Antonine Physics A2

ANTONINE EDUCATION

Topic 14A Astrophysics

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

The study of the stars has been fascinating for scientists for many centuries, and new discoveries today cause as much excitement for astronomers as they did thousands of years ago. It also captures the imagination of the public; astronomy and astrophysics are seldom far from the news headlines.

Astronomy and astrophysics are based on sound physics principles that apply in the here and now on Earth every bit as much as they do in space. Newton's and Keppler's Laws explained the movement of astronomical objects and are valid three hundred years later.

Daniel Weadley runs another basic astronomy guide, Astronomy for Kids. See it HERE

For all the latest in space news from NASA, go to https://www.space.com/

Alex MacColgan has an excellent YouTube channel called **Astrum**.

Topic 14 A

Option A Astrophysics

1. Telescopes

Tutorial 14A.01 Lenses and Refracting Telescopes

AQA Syllabus

Contents

14A.011 Introduction

The night skies have fascinated people for many thousands of years. The earliest scientific observations were of the movements of stars and planets. The term planet comes from the Greek word "planein" - to wander. They were wandering stars. The wandering movement is now explained by the idea of the planets orbiting the sun. Our fascination with the stars and planets has not diminished even though we know a lot more about the Universe now than we did before.

Astrophysics is about the application of physics principles to explain how the Universe works. Here are some terms that astronomers and astrophysicists use to describe bodies in the universe:

- **Satellite** - a smaller object that orbits about a larger object.
- **Star** - an object that gives out radiation by fusion reactions. Some large gas planets give out radiation because they are hot.
- **Planet** - an object that orbits a star.
- **Solar system** - several planets that orbit a star.
- **Constellation** - a collection of stars that form a pattern recognisable from the Earth. These are often named after mythical beings that were important in ancient cultures.
- **Galaxy** - a group of many hundreds of thousands of stars. Astrophysicists believe that most (if not all) galaxies have a large black hole in the middle.
- **Universe** - a term that takes in all objects and materials in space, seen and unseen.

New discoveries are being made all the time. Current theories about the way the Universe works are being constantly tested and update. Some are being re-written.

Do NOT use the term "astrology" to describe any aspect of astronomy or astrophysics.

Astrology is a system of superstitions that claims to be able to predict events in the future by the movements of stars. Columns are found in the popular tabloid newspapers that claim that "today is a good day for financial speculation".

The publicity for A-level Physics for the college I used to work for was marred by the inclusion of "astrology" as an option. I complained several times to the marketing department. Despite the involvement of the Principal, the howler remained.

14A.012 Convex Lens

Lenses work by **refracting** light at a glass-air boundary. Although refraction occurs at the boundary, we will treat all lenses as bending the rays at the **lens axis**.

The lens in the eye is a **convex** or **converging lens**. This means that the lens makes rays of light come together or converge (*Figure 1*).

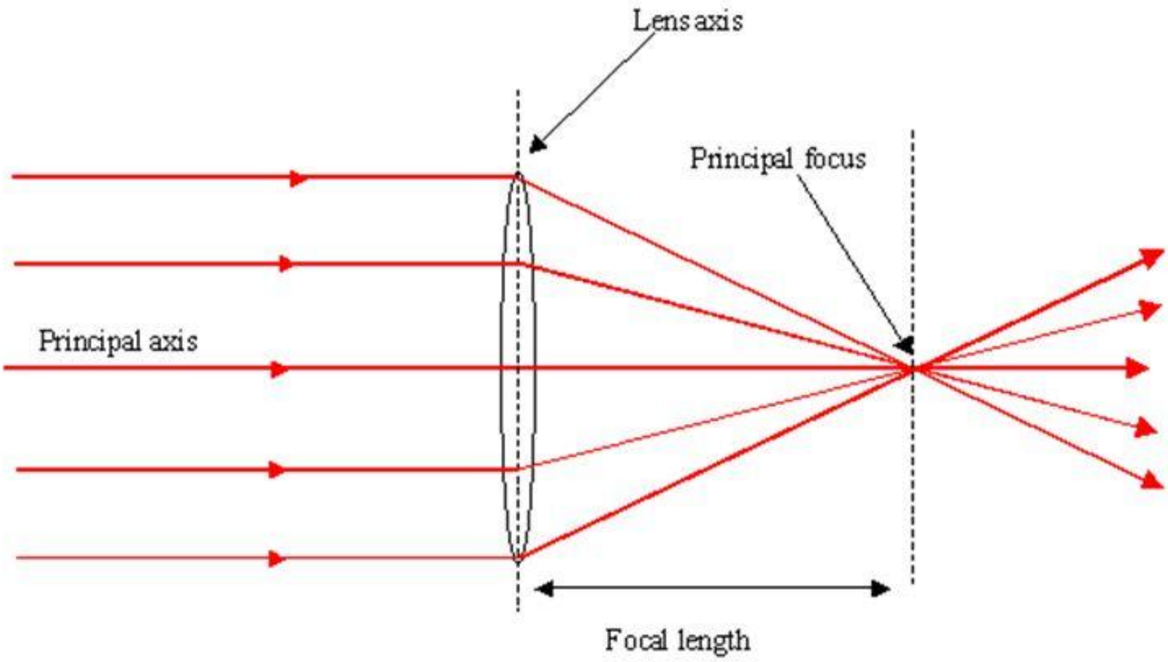

Figure 1 Parallel rays of light passing through a convex lens

The rays parallel to the **principal axis** are converged onto the **principal focus**. The **focal length** is the distance between the **lens axis** and the **principal focus** (strictly speaking, the focal plane). The focal length is given the code F.

Thicker lenses bend light more and are therefore described as more powerful. Powerful lenses have short focal lengths. The power of a lens is measured in **dioptres** (D) and is given by the formula:

$$\text{Power} = \frac{1}{\text{focal length (m)}}$$ …………… *Equation 1*

The powers (in dioptres) of several thin lenses add up so that:

$$P_{\text{tot}} = P_1 + P_2 + P_3 + \ldots$$ …………… *Equation 2*

The principal focus of a convex lens is called **real**. The images made by convex lenses are in most cases real. This means that the image can be projected onto a screen. We will see later how images are made with **ray diagrams**.

Principal focus, not principle focus. (**Principal** means *main*, or *chief*; **principle** means *rule*.)

14A.013 Ray Diagrams

We can determine where an image lies in relation to the objects by using a **ray diagram**. We can do this by using two simple rules:

- Draw a ray from the top of the image parallel to the principal axis. This ray bends at the lens axis and goes through the principal focus.
- Draw a ray from the top of the lens through the centre of the lens.

Where the two rays meet, that is where the image is found. In this example, we have placed the object between F and $2F$. The diagrams show how we do a ray diagram step-by-step:

Step 1. Draw the ray parallel to the principal axis (*Figure 2*).

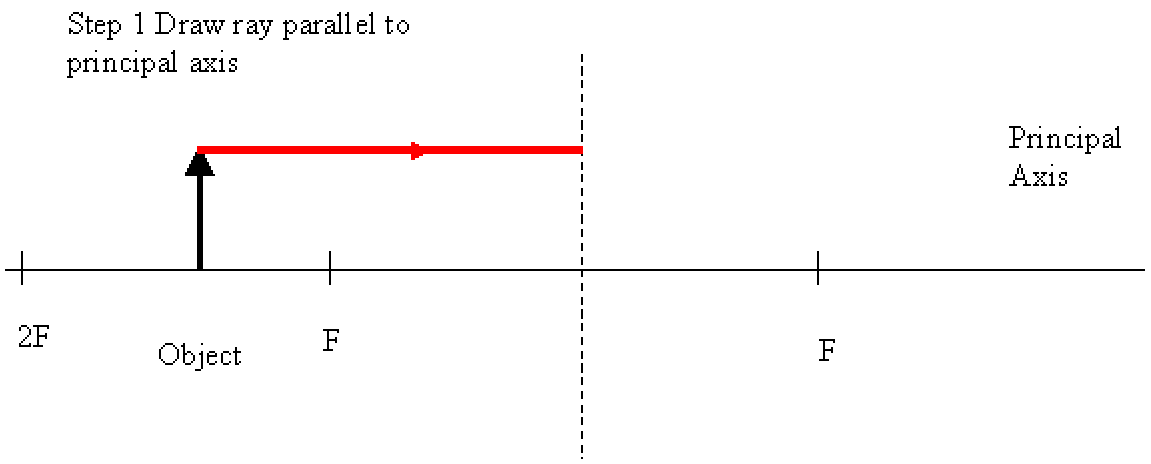

Figure 2 Drawing a ray parallel to the principal axis

Step 2. Draw the refracted ray so that it passes through the principal focus (*Figure 3*).

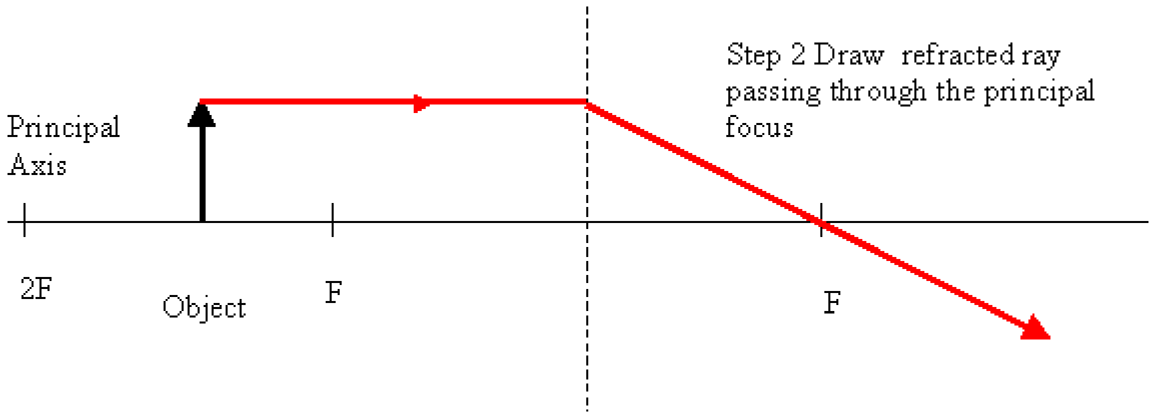

Figure 3 Drawing a line between the lens axis and the principal focus

Step 3. Draw a ray from the top of the object through the middle of the lens. This ray is **undeviated** (*Figure 4*).

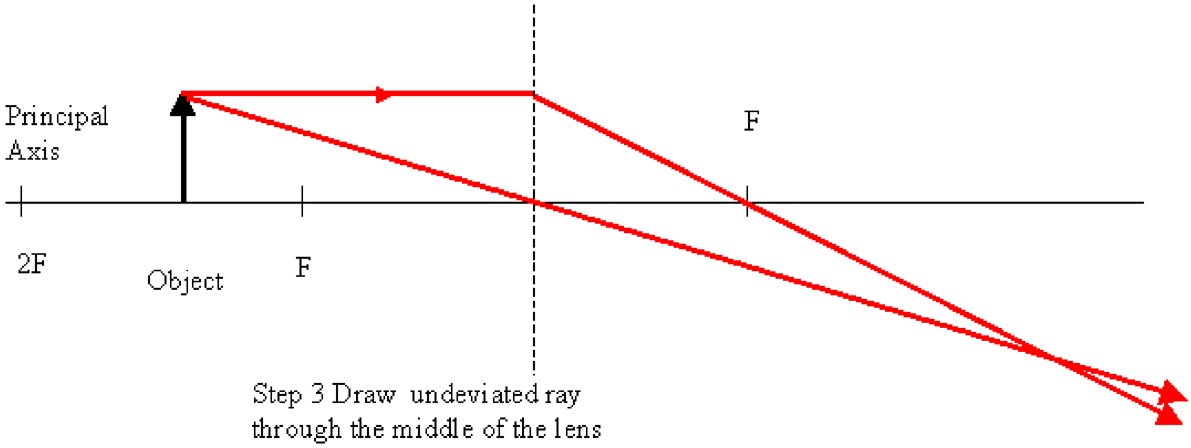

Figure 4 Adding the undeviated ray

Step 4. Where the rays meet, that is where the image is (*Figure 5*).

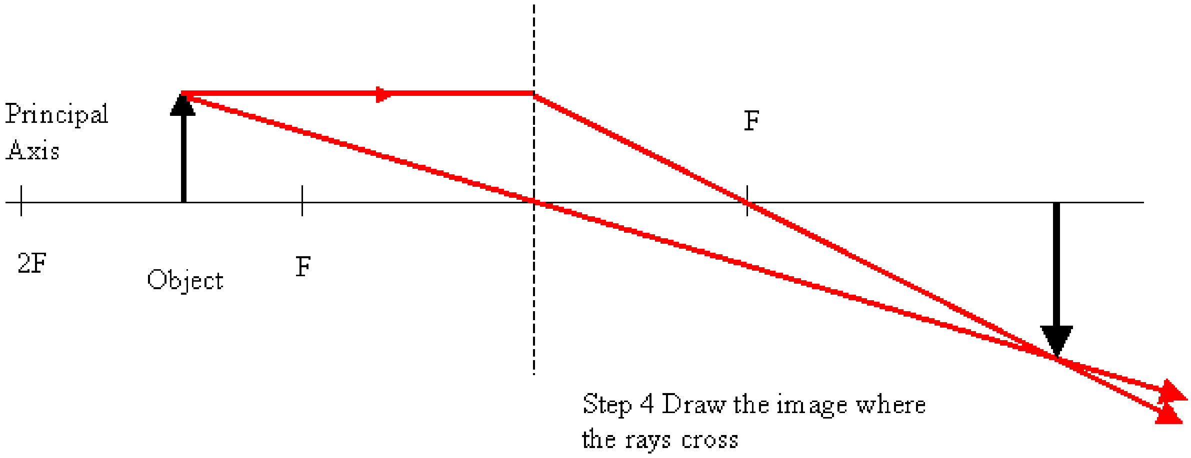

Figure 5 Final diagram to find the image

It is a good idea to draw your ray diagrams on graph paper as the following ray diagrams are. Be careful with your drawing; a small change in the angle of the undeviated ray can lead to quite a big change in the final position of the image. And PLEASE… Be a good chap and use a sharp pencil. The image is **inverted** (upside down), **real**, and **magnified** (bigger).

Here is a ray diagram done on graph paper (*Figure 6*).

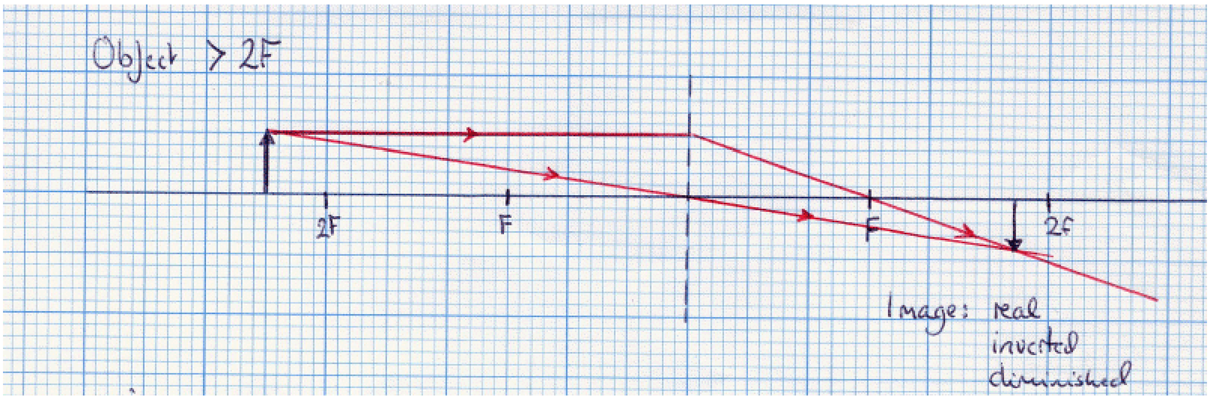

Figure 6 Ray diagram drawn on graph paper

This diagram shows where an object is at a distance of greater than twice the focal length. The image is **inverted** (upside down), **real**, and **diminished** (smaller).

14A.014 The Lens Formula

Lens diagrams have the main disadvantage that there is uncertainty in precisely where the image is. Therefore, the use of the lens formula is better. The lens formula is:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

.... *Equation 3*

[f - focal length (m); u - object distance (m); v - image distance (m)]

Worked Example An object of height 1.60 cm is placed 50 cm from a converging (convex) lens of focal length 10 cm. What is the position of the image?
Answer Substitute: $$\frac{1}{10\text{ cm}} = \frac{1}{50\text{ cm}} + \frac{1}{v}$$ $$\frac{1}{v} = 0.10\text{ cm}^{-1} - 0.020\text{ cm}^{-1} = 0.080\text{ cm}^{-1}$$ $v = (0.080\text{ cm}^{-1})^{-1} =$ **12.5 cm**

It does not matter if you work in cm, as long as you are consistent. However, if you are going to use dioptres you must work in metres.

The **magnification** is worked out using this simple formula:

$$M = \frac{v}{u}$$

.... *Equation 4*

Since v is in metres, and u is in metres, M has no units.

Worked example What is the magnification in the example above? What is the size of the image?
Answer M = 12.5 cm ÷ 50 cm = 0.25   Image height = 1.60 cm × 0.25 **= 0.40 cm** = 4.0 mm

The convention for the equation is that **real is positive**. For a concave lens, the focal length is negative, because the principal focus is virtual. If the image position gives a negative value, then the image is **virtual**.

14A.015 The Telescope

In this section we will look at the **refracting** telescope works by bending light with lenses. The **objective** lens makes a small real image of the object while the **eyepiece** lens acts as a magnifying glass. The following factors are important in making a good quality instrument:

- Lens quality: bad lenses, bad image.
- lens diameter: brightness and detail observed depend on the area. A 12 cm lens can resolve detail nine times better than a 4 cm lens.
- Angular magnification.

The diagram (*Figure 7*) shows the telescope when it is set up normally (**normal adjustment**).

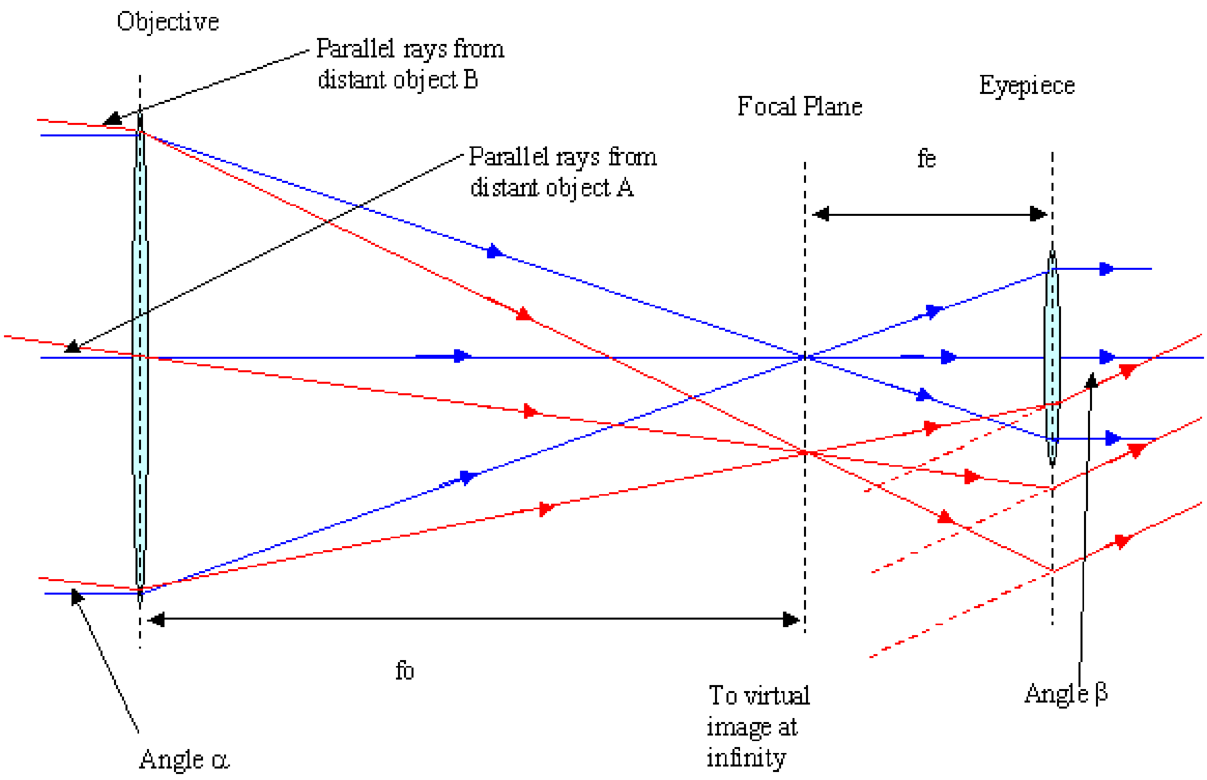

Figure 7 Ray diagram of a refracting (lens) telescope

Light from object A (blue lines) meets at the **principal focus** of the objective lens. It then spreads out until it meets the eyepiece. The eyepiece is set at the focal length away from its **principal focus**. Parallel rays emerge from the eyepiece.

At the same time parallel rays from object B arrives at the objective at a small angle a to the axis. The light is focused onto the **focal plane**. It then passes through the eyepiece to emerge as parallel rays. The angle of these parallel rays is b to the parallel rays from A.

The **angular magnification** can be worked out by the simple formula:

$$M = \frac{\beta}{\alpha}$$ .... *Equation 5*

where α and β are small angles in **radians**.

- The angle α is the angle subtended by the object to the unaided eye.
- The angle β is the angle subtended by the image to the eye.

The magnification can also be shown to be related to the **focal lengths of the lenses** by:

$$M = \frac{f_o}{f_e}$$ .... *Equation 6*

The telescope shown above makes an **inverted** image. To make the image the right way up we need to put in a third lens at the principal focus of the objective lens, but we won't go into that at this point.

A big problem with the refracting telescope is **chromatic aberration**. Different colours refract by different amounts. You can see this in poor quality telescopes, in the form of spectra around the edges of the lenses.

14A.016 Brightness of the Image

When you look at a star with a telescope, the star seems brighter. This is because the area of the telescope lens is much bigger than the area of your pupil. Brightness is due to the **intensity** of the light, or power per square metre. The brightness goes up proportionally with the area, or proportionally with the **square** of the diameter. If your eye is 1 cm in diameter, a telescope with a lens 5 cm in diameter will give an image that is 25 times brighter. If the lens is 15 cm in diameter, the intensity will be 15^2 = 225 times more.

Questions

Tutorial 14A.01

14A.01.1

The power a lens is +0.2 D. What is the focal length in metres?

14A.01.2

What is the image like if the object is at $2F$?

14A.01.3

What is the image like if the object is between $2F$ and F?

14A.01.4

What is the image like if the object is at F?

14A.01.5

What is the image like if the object is less than F?

14A.01.6

Find the position and size of an old pound coin, 2.2 cm in diameter placed 20 cm from a converging lens of focal length 40 cm.

What are the properties of the image?

14A.01.7

In a telescope the eyepiece has a focal length of 2 cm, and the objective has a focal length of 220 cm. What is the magnification? If the moon subtends an angle of 8.8×10^{-3} rad to the naked eye, what would the angle be for the image of the moon observed through the telescope?

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AQA Syllabus	
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14A.023 Diffraction Effects	14A.024 Rayleigh's Criterion
14A.025 Derivation of the Equation	14A.026 Light Detectors

14A.021 Reflecting Telescopes

Lenses suffer a major drawbacks:

- They refract light of different colours by different amounts. You can see that when light is split into a spectrum with a prism. This leads to **chromatic aberration**. The image is distorted.
- They do not transmit 100 % of the light; some is lost.
- Large lenses are very difficult to make.
- To get a good magnification you need an objective lens with a very long focal length. This can make the telescope very long. The largest is 20 m, with a 1 m wide objective lens.

Concave mirrors can be used to project a real image (*Figure 8*):

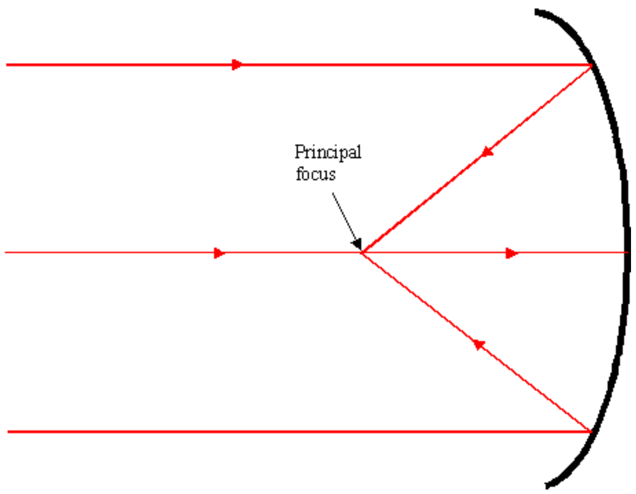


Figure 8 A concave mirror projecting a real image

The advantages of a concave mirror that is **front-silvered** are:

- it can be made bigger (large diameter lenses are very hard to make);
- there is no chromatic aberration in reflection.

The focal length of a concave mirror is half the radius of curvature. Spherical mirrors are easy to produce, but the image can be distorted by **spherical aberration** (*Figure 9*) so a **parabolic** shape is used to give perfect focusing (*Figure 10*). You may wish to review curved mirrors in Topic 7.

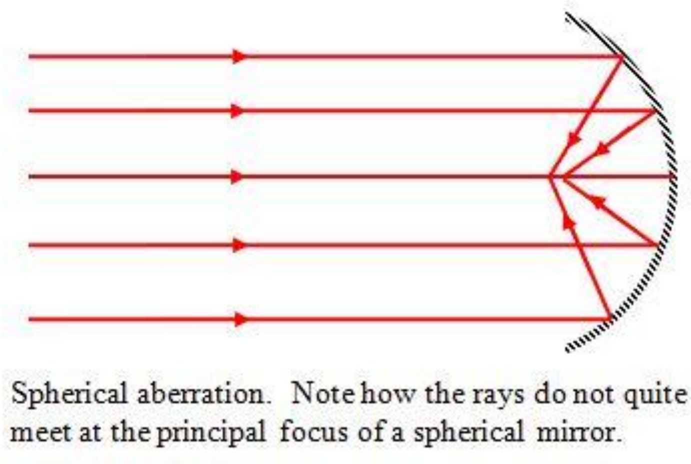


Figure 9 Spherical aberration

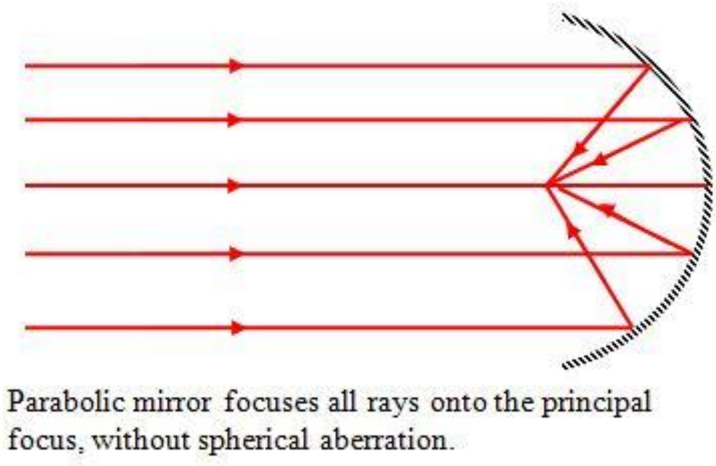


Figure 10 A parabolic mirror removes spherical aberration

Many other receivers of electromagnetic radiation use parabolic mirrors.

The diagrams show the two kinds of reflecting telescope (*Figures 11* and *12*):

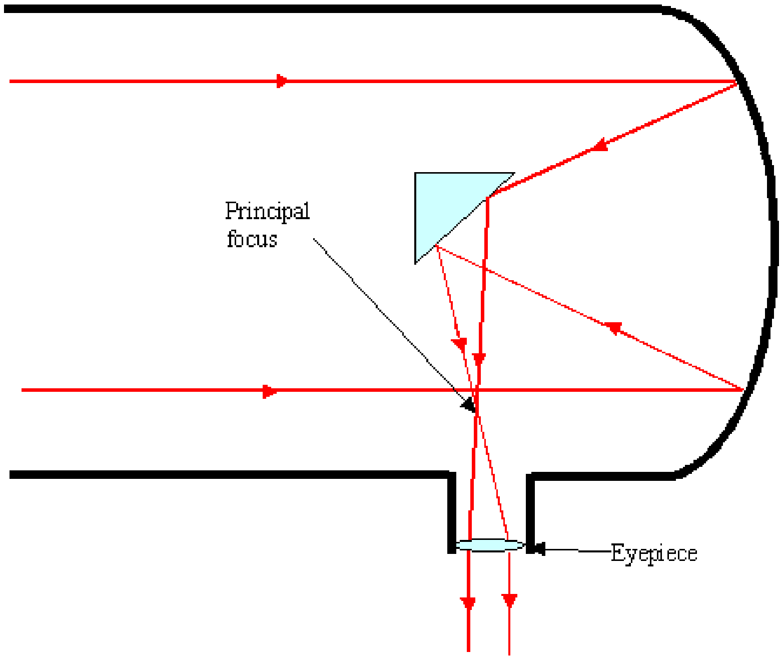


Figure 11 A Newtonian reflecting telescope

This is called the **Newtonian system**. Light is reflected to an eyepiece at the side of the telescope.

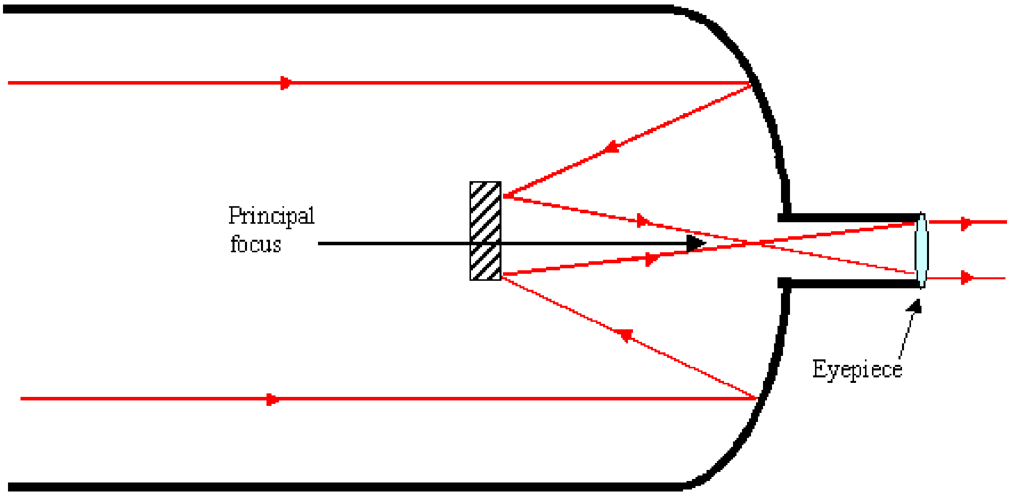


Figure 12 Cassegrain reflecting telescope

This telescope uses the **Cassegrain system**. The eyepiece is at the back of the telescope. The hole in the centre of the mirror does not affect the viewing ability. Both kinds are found in observatories. The mirror shown in this diagram is a plane mirror. However, many instruments use a **convex mirror** as the secondary mirror. Some radio telescopes also use the Cassegrain system of mirrors.

All large telescopes use the reflecting system. The largest telescope in the world has a 5 metre diameter concave mirror which requires many tonnes of glass, a considerable cooling time, and many hundreds of hours of grinding to get it to a perfect shape. It was silvered with a few grams of aluminium.

14A.022 Resolving Power

The **resolving power** of any optical instrument is an indication of how good it is at distinguishing two objects close to one another. For example, at a long distance two car headlights appear as a single blob of light. At about 5 km, we can tell that they are two separate lights.

This is because the eye can resolve down to an angle of about **3×10^{-4} radians**.

Astronomers use angles in **radians** or **degrees**:

- There are 2π radians in a circle = 360°.
- 1 rad is approximately 57°.
- Degrees are subdivided into **arc-minutes** (1° = 60 arc-minutes [60'])
- and **arc-seconds** (1' = 60 arc seconds [60''])

Radians have the advantage that for small angles:

$$\sin\theta = \tan\theta = \theta$$ ………… *Equation 7*

This makes trigonometrical functions easier. However, astronomers tend to use arc-seconds rather than radians which are useful for describing very small areas of sky. In this case, *Equation 7* cannot be used.

The observation of objects in space is made difficult because the atmosphere is turbulent. This results in the twinkling or **scintillation** of stars. **Light pollution** from street lights does not help either. Dust in the atmosphere causes **scattering** of light.

Major observatories have moved as far away as possible from cities and are situated on high mountains.

Figure 13 The Hubble Space Telescope (Photograph from NASA)

The best images of them all come from the Hubble Telescope (*Figure 13*), a Cassegrain instrument which is in orbit above the Earth. There are no problems with distortion of the atmosphere up in space, but doing routine maintenance is not very easy. The quality of pictures produced has been very high (*Figure 14*).

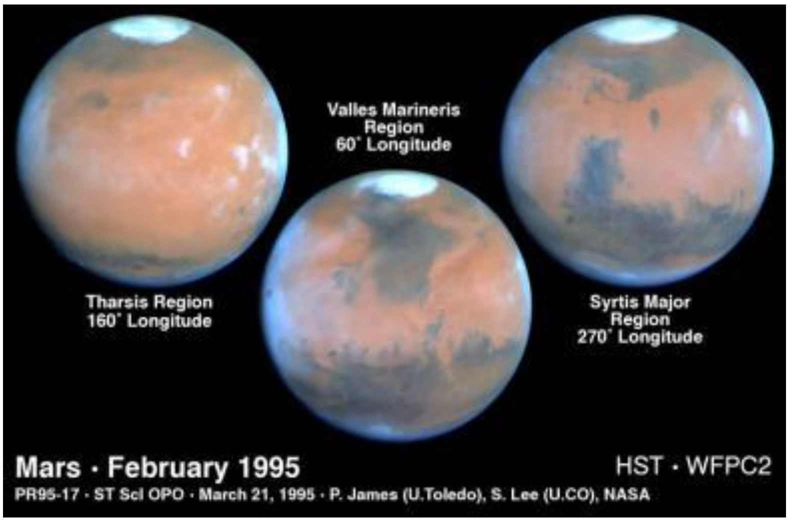


Figure 14 High quality photographs of Mars (Photograph from NASA)

14A.023 Diffraction Effects

When light enters a telescope, it is passing through a gap. It spreads out by the process of diffraction. You will remember from AS Waves how when light passes through a single slit (*Figure 15*), dark and bright fringes are made. (You might want to break off and revise that bit. Go to Topic 7.) The resulting pattern is called a Fraunhofer Diffraction pattern (*Figure 16*).

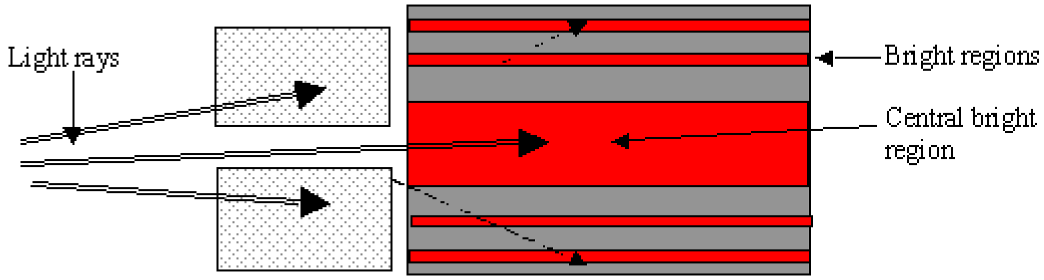


Figure 15 Passing light through a single slit

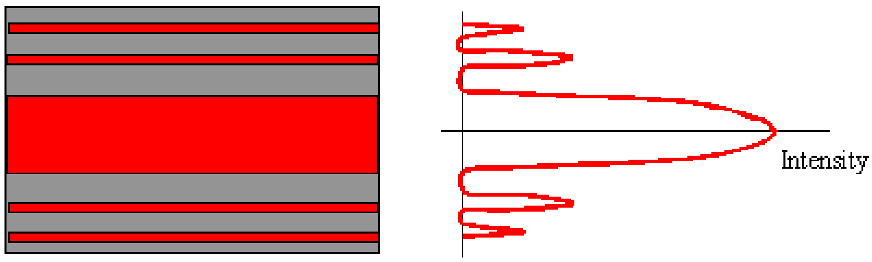


Figure 16 Frauenhofer diffraction

Fraunhofer diffraction also occurs with circular openings. If we use a circular aperture, we get an effect like this (*Figure 17*).

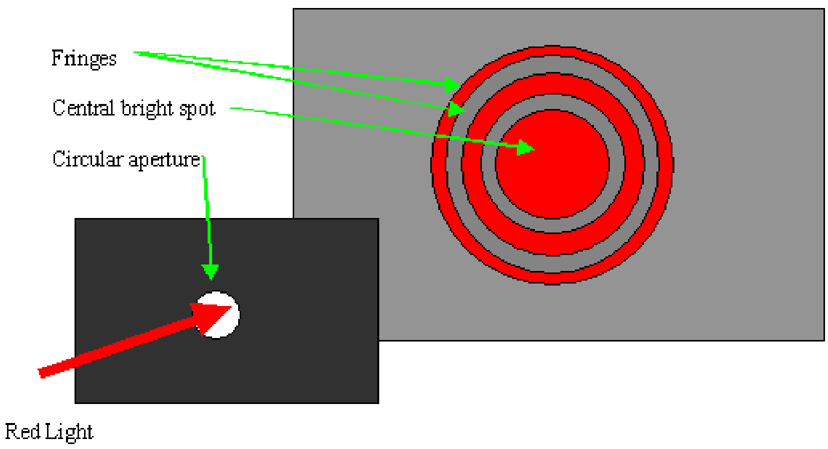


Figure 17 Frauenhofer diffraction with a circular aperture

The central bright spot is called an **Airy Disc**. The telescope can be modelled as a single circular hole. Diffraction is inevitable, but the larger the hole, the narrower the diffraction pattern becomes. (Conversely a narrow aperture leads to a greater diffraction pattern.

14A.024 Rayleigh's Criterion

Consider two stars A and B. They are separated by an angle of θ radians (*Figure 18*).

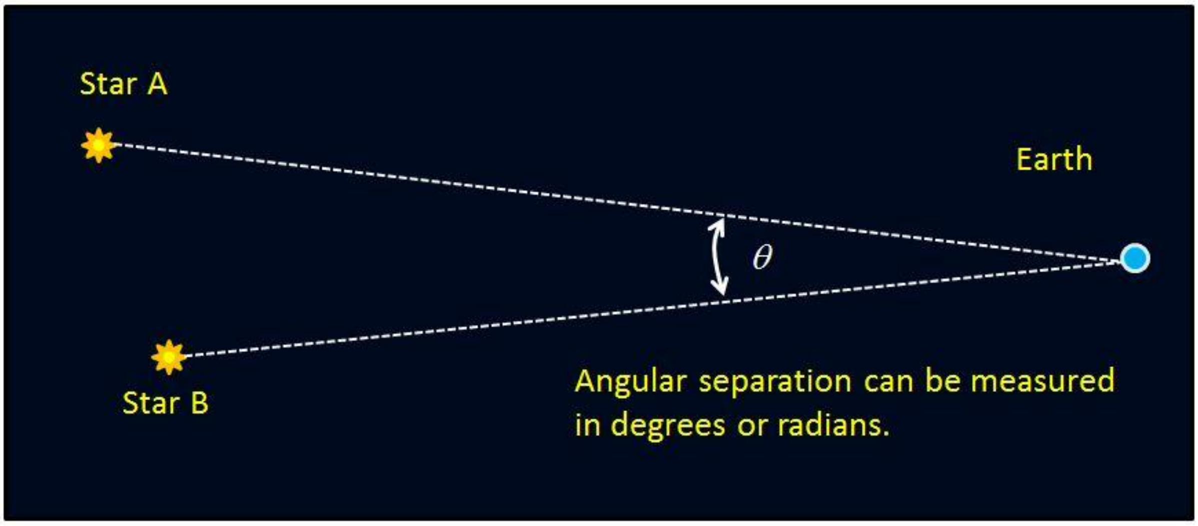


Figure 18 Two stars separated by an angle of θ radians

If the angle θ gets smaller, the two stars seem to merge into one. Therefore, we need a more powerful telescope to observe them. The physicist John William Strutt, Lord Rayleigh (1842 - 1919) studied the effect of overlapping of fringes and came up with the **Rayleigh's Criterion**.

Rayleigh's criterion says:

The resolution of the images of two point objects is not possible if any part of the central spot of either image lies inside the first dark ring of the other image.

This is shown in *Figure 19*.

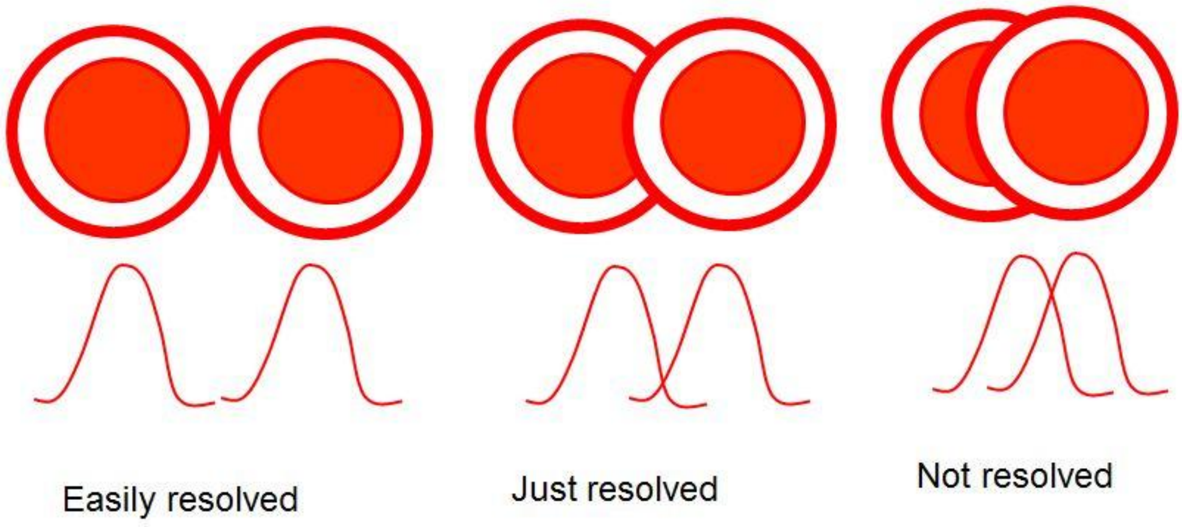


Figure 19 Resolution of objects using Rayleigh's criterion

The angular separation at which the two objects are resolved is given by the formula:

$$\theta = \frac{\lambda}{D}$$
...............*Equation 8*

[θ - angular separation (rad); λ - wavelength (m); D - aperture width (m)]

The angle term θ must be in radians. This is because $\sin\theta \approx \theta$ in radians. If the angle is in degrees, the relationship becomes:

$$\sin\theta = \frac{\lambda}{D}$$
.... *Equation 9*

Where:

$$\theta < \frac{\lambda}{D}$$
.... *Equation 10*

the two stars cannot be resolved. To improve the resolution of a telescope, we need to have a large aperture and a short wavelength.

In the AQA syllabus, the aperture is regarded as a **single slit**. In other syllabuses, the aperture is regarded as a **circular disc**. In which case, the relationship is modified to:

$$\theta = 1.22\frac{\lambda}{D}$$

.... *Equation 11*

In practice, although telescopes have much better resolution than the eye, this is limited by the atmosphere. Telescopes have large apertures to allow as much light to get in as possible.

14A.025 Derivation of the Equation

We know that the diffraction equation for a diffraction grating is:

$$n\lambda = d\sin\theta$$

.... *Equation 12*

Where:

- n = number of orders
- λ = wavelength (m)
- d = slit spacing (m)
- θ = angle (rad)

In this case, n = 1 and d = diameter of the telescope in m. Also, for a small angle in radians, $\sin\theta = \theta$.

So, we can now write:

$$\lambda = d\theta$$

.... *Equation 13*

And we can rearrange to give:

$$\theta = \frac{\lambda}{D}$$

.... *Equation 14*

Angle θ must be in radians. Make sure that your calculator is set to radians.

14A.026 Light Detectors

Once we have got a good image down the telescope, we need to have a way of recording it. In early astronomy the human eye was used, and the results depended on the artistic ability of the astronomer. Photographic techniques were used from the middle of the Nineteenth Century.

The resolution depends not just on the Rayleigh Criterion, but also on the **emulsion** of the film. Very fine grain films are used for astronomical observation. The quality of the picture needed to be high and precision mechanisms were essential for **tracking** individual stars across the sky. If the grains of film are larger than the resolution of the telescope, than that is the limiting factor.

For many years, photographic film was the only way to record images of telescopes. The quality of the images using wide format film could be outstanding. Getting good quality images requires:

- good quality optics.
- that a film needs to be exposed for a long period of time.

This is because the film needs photons to deposit the silver grains. The film has to be very fine-grained, to give it the quality of image. This makes the film slow. Therefore, the telescope needs to track across the sky in order to follow the object under study.

Another problem with photographic film is that it has to be chemically processed. While it's a straight forward process, it has been known to go wrong. If the film consists of a number of exposures that have taken hours on cold nights to expose…

More recently a **charged coupled device** is used and is connected to a computer. The computer can quickly do comparisons of images which would take a skilled astronomer several days. The CCD works like this, as shown in this animation (*Figure 20*). You are NOT expected to know how it works.

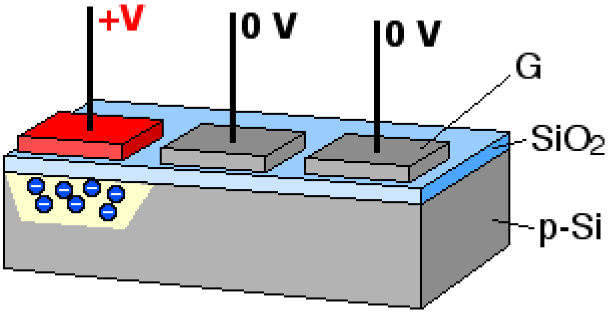


Figure 20 Animation of a CCD (Animation by Michael Schmid, Wikimedia Commons)

The picture below shows a CCD in a camera (*Figure 21*).

Figure 21 CCD in a camera (Kaspar Metz, Wikimedia Commons)

The CCD is about the size of a postage stamp and can have many millions of pixels on it. They work on the principles of **quantum physics**. They have these advantages over film:

- They are sensitive.
- Their quantum efficiency is about 70 %. A film has a quantum efficiency of about 4 % which means that 25 photons are needed to deposit a grain of silver.
- They are getting cheaper all the time.
- The CCD can detect radiations that are beyond the visible spectrum.

The graph below (*Figure 22*) shows the quantum efficiency of the CCD:

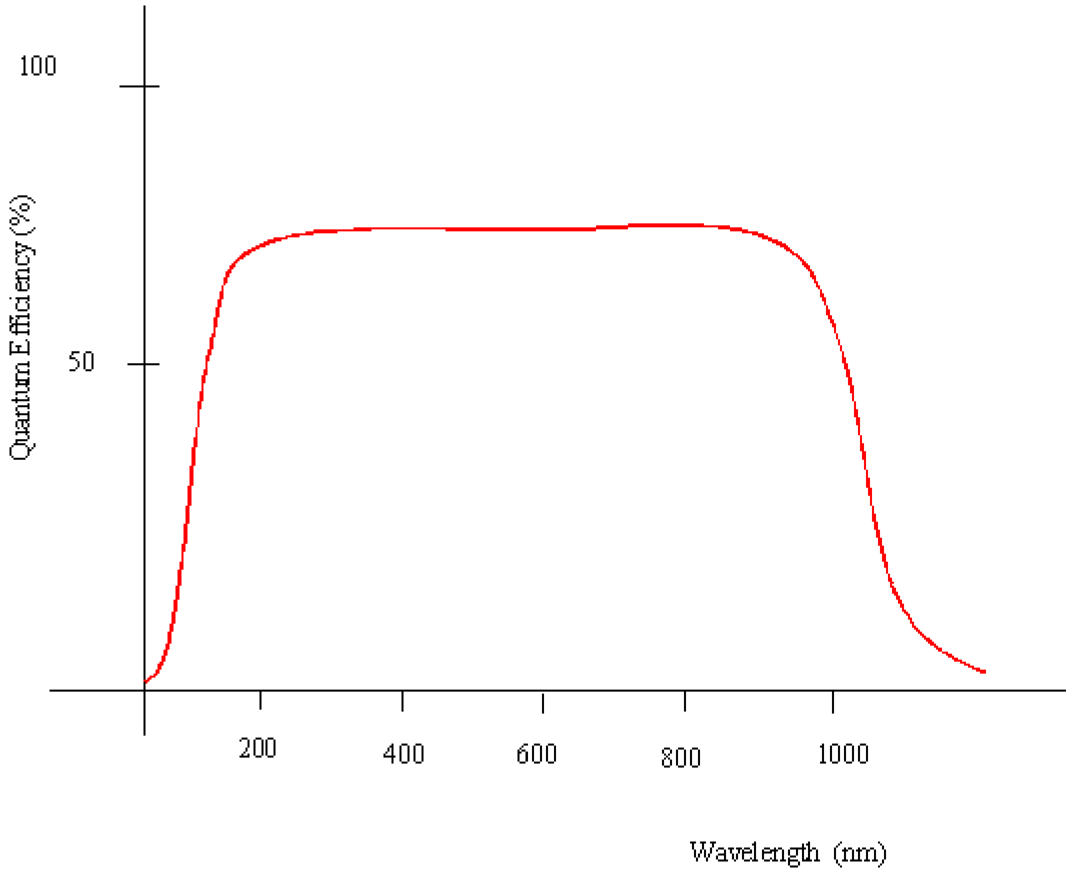


Figure 22 Graph of quantum efficiency against wavelength

The eye has a quantum efficiency of only 1 %.

With a CCD, the image is uploaded to a computer, where software is used to combine the images to give a colour picture. The pictures can be of stunning quality.

CCDs can detect infra-red radiation. The **James Webb Space Telescope** has a 6.5 m diameter mirror that focuses infra-red radiation onto a CCD to explore the IR characteristics of a range of very distant objects in the universe.

Questions

Tutorial 14A.02

14A.02.1

Suggest reasons for the following:

(a) The silvering on a telescope mirror is on the top surface.

(b) The hole in the centre of the mirror of the Cassegrain system does not affect the viewing ability of the instrument.

14A.02.2

The Moon has a diameter of about 3500 km and is about 400 000 km from the Earth. What is the angle in radians that the Moon subtends to an observer on the Earth? What is this in degrees?

14A.02.3

Entirely coincidentally the angle subtended by the Sun is exactly the same as the angle subtended by the Moon. The distance between the Earth and the Sun is 150 x 10^6 km. What is the diameter of the Sun?

14A.02.4

What is the resolving power of a telescope of diameter 15 cm at a wavelength of 600 nm?

Tutorial 14A.03 Radio Telescopes

AQA Syllabus

Contents

14A.031 The Radio Telescope

The **radio telescope** was first devised during the 1930s. During the Second World War, considerable advances were made with all sorts of techniques used for radio for communication, detection, and navigation. Sophisticated apparatus was made for eavesdropping on enemy radio traffic to gain intelligence. (The Germans were excellent soldiers but never appreciated the need for good intelligence; the Allied Powers were very good at intelligence and deception.)

The initial driving force for RADAR (Radio detection and ranging) was originally to make a radio death-ray to make bombers fall out of the sky in flames! The initial driving force for radio telescopy was to listen out for radio signals from other extra-terrestrial civilisations (little green men). The first **pulsar** was discovered by Jocelyn Bell Burnell in 1967. It was called LGM-1 (Little Green Men 1 – yes; it was called that).

In Britain, the first work to be done with radio telescopes was carried out by Professor Sir Bernard Lovell (1913 - 2011) and a team from the University of Manchester. They set up war-surplus radio equipment bought from the army, in a field in the middle of Cheshire. The work grew and in the middle nineteen fifties the team undertook the construction of a massive radio telescope, which is shown below (*Figure 23*). (Sir Bernard was also a very accomplished organist, outside his role as an astronomer.)

Figure 23 Jodrell Bank radio telescope (Photo by Mike Peel, University of Manchester)

This massive instrument at **Jodrell Bank** is 75 m across. Its story is major work in itself, but it eventually started exploration in the late nineteen fifties, almost exactly at the same time as electric trains powered by 25 000 V overhead power lines started running on the main line that passes 200 m from the instrument.

- The **dish** is **parabolic**, reflecting radio waves onto an **antenna** at the **principal focus**. The radio waves are very weak, and the focusing by the reflector makes them much more intense.
- The diameter of the dish is sometimes called the **objective diameter**.
- The antenna detects the intensified radio waves.
- The **receiver** has to be tuned in, just like any other radio set.
- The signal is passed down to very high quality amplifiers, and the signals are **analysed** by a computer.
- Like a light telescope, the instrument has to be **tracked**, otherwise the object will be lost.
- The **power** of the telescope is proportional to the **square of its diameter**.

The largest radio telescope in the world is the Arecibo telescope in Puerto Rico. It is built between some small hills that had a roughly parabolic valley. It is 300 m across. The Puerto Rico Instrument has the valley floor paved in metal sheeting to act as the mirror.

Recent developments have concentrated on producing **arrays** of radio telescopes. Instead of one huge dish, there are many smaller instruments that move in unison (at the same time) and point at the same area of the sky. The effect of this is the same as having one huge dish, without all the difficulties of making one. The picture (*Figure* 24) shows the idea:

Figure 24 Array of radio telescope antennae (Computer generated picture by SKA Project Development Office and Swinburne Astronomy Productions, Wikimedia Commons)

This is a current project called the **Square Kilometre Array**. As its name suggests, the area of each array will be 1 km^2. There will eventually be several of these.

14A.032 Resolving Power

High resolution enables us to see things in good detail (*Figure 25*).

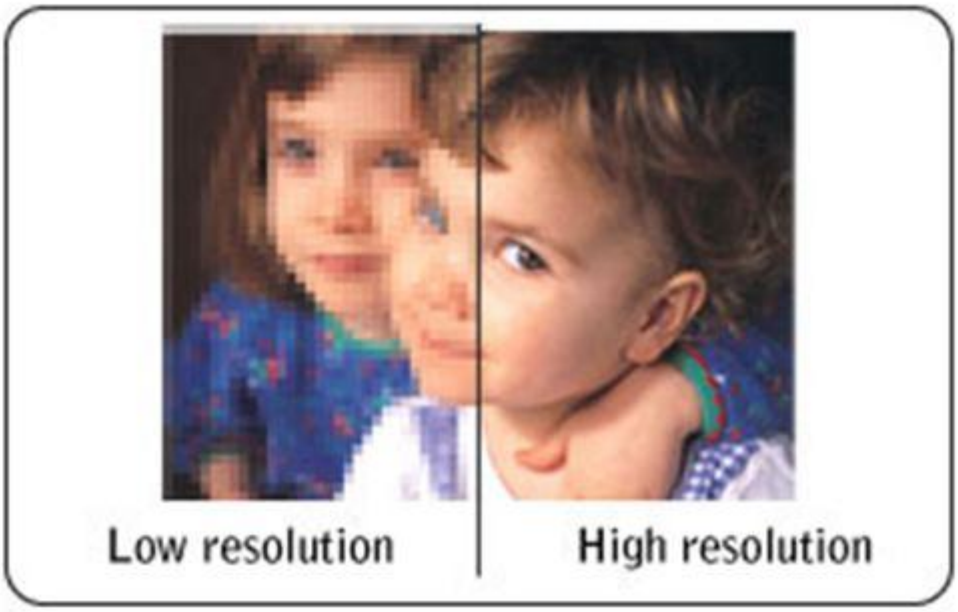


Figure 25 Low and high resolution

The girl is low resolution; she is fuzzy and pixellated. You can see more detail on the boy. The higher the resolution, the smaller the **angle of separation** that can be observed. The angular separation of two stars, A and B, is shown below (*Figure 26*):

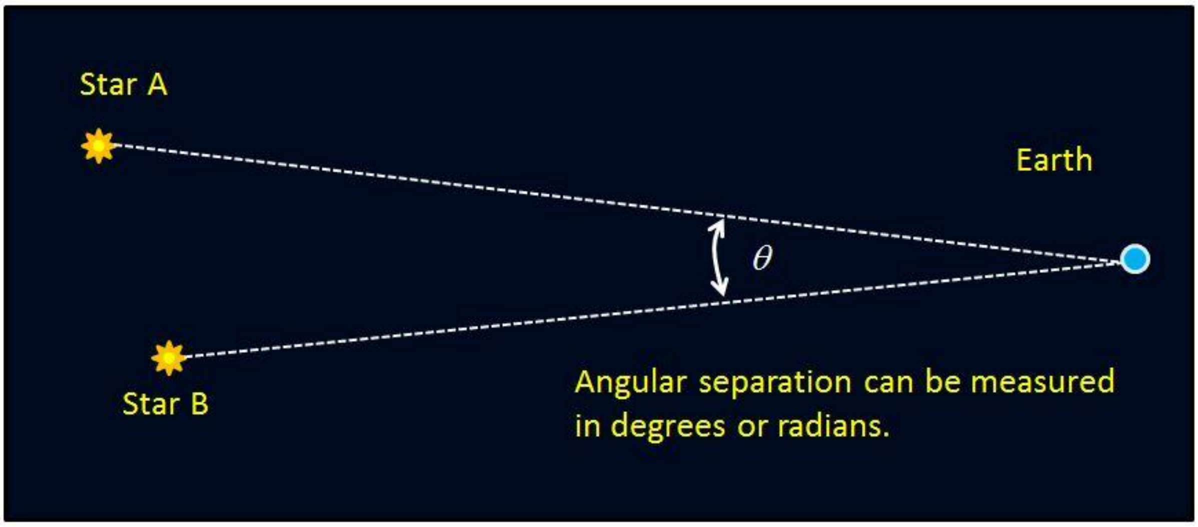


Figure 26 Angular separation between two stars

If the two stars are viewed through a telescope, **diffraction** will occur. This could blur the image.

The **resolving power** of a radio telescope is governed by the same kind of factors as a light telescope. It is given by Rayleigh's Criterion:

$$\theta = \frac{\lambda}{D}$$

.... *Equation 15*

From your answer to Question 14A.03.4, you can see that the resolution is not very good. To resolve between radio sources, the telescope has to scan across to detect the precise origin of each source.

The dish does not have to be as perfect as mirror for a light telescope. As long as the surface is within about **1/20 wavelength**, then the focusing will be unaffected by imperfections. Also, the reflector does not have to solid. **Fine wire mesh** will do, since radio waves will not pass through a gap less than one wavelength.

14A.033 Radio Sources in the Universe

Radio astronomy has:

- revealed the existence of radio sources, such as **quasars** and **pulsars**.
- been used to **analyse chemical elements** in objects.
- tracked the **movement of planets** using the Doppler effect.
- looked at **microwave radiation** which gives evidence for the big bang.

The picture below (*Figure 27*) shows the same region using **far infra-red** compared with the sky seen with visible light. Far infra-red is at the same area of the EM spectrum as microwaves.

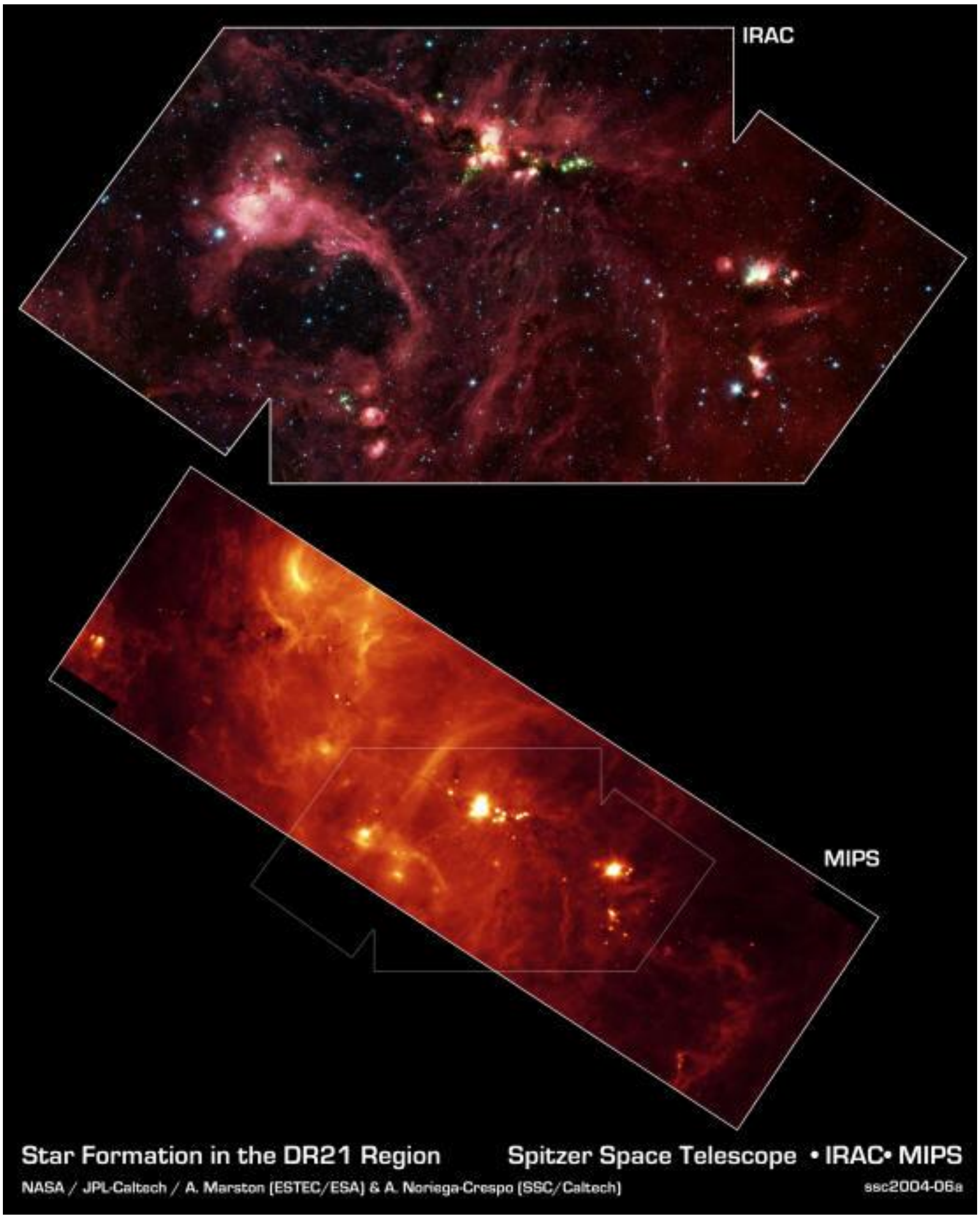


Figure 27 Comparing a visible light image (IRAC) to a far infra-red image of the same region (NASA)

Radio waves can penetrate dust, so we can look at the centre of our galaxy. However, radio waves of wavelength less than about 1 cm are blocked out by carbon dioxide and water. Radio waves of wavelength 20 m and above are absorbed by the atmosphere. Also, radio signals from Earth can cause interference, just like light pollution for light telescopes. Passing satellites can also obscure the field of view.

Many radio telescopes have been set up well away from cities. Satellites with radio telescopes have been used to investigate the microwave radiation that points to the Big Bang.

14A.034 Other kinds of Telescope

The diagram above shows that telescopes can be made to look at a large range of different electromagnetic radiations:

- infra-red.
- UV light.
- X-rays.
- gamma rays.

The next picture (*Figure* 28) shows the same patch of sky visible to all sorts of different wavelengths.

Figure 28 Sames patch of sky when viewed with different radiations (NASA)

Infra-red telescopes allow astronomers to observe objects that are too cool to give out visible light. These include cooling dead stars, planets, and dust clouds. A difficulty with infra-red telescopes is the fact that on the ground, the surface is irradiating the instrument with infra-red all the time. Therefore, the dish has to be cooled, or the infra-red would swamp what is being observed. It's a bit like trying to observe a star behind the Sun. Even worse is that water vapour absorbs and retransmits infra-red. The latter problem can be reduced by sending a satellite into orbit. However, the receiver has to be cooled to a few Kelvin to observe very weak sources.

Infra-red sources give wavelengths of about 700 nm to 1 mm.

UV Telescopes

These will not work on the Earth's Surface as UV light is absorbed by the Earth's atmosphere. So, they are sent up into orbit. Glass lenses absorb UV, so mirrors are used to focus the rays onto a charge-coupled device. The data are transmitted digitally to be interpreted by appropriate software. UV is emitted by materials at high temperatures. Objects can be looked out using a variety of detectors, and the UV light shows hot spots. Here is such an image (*Figure 29*).

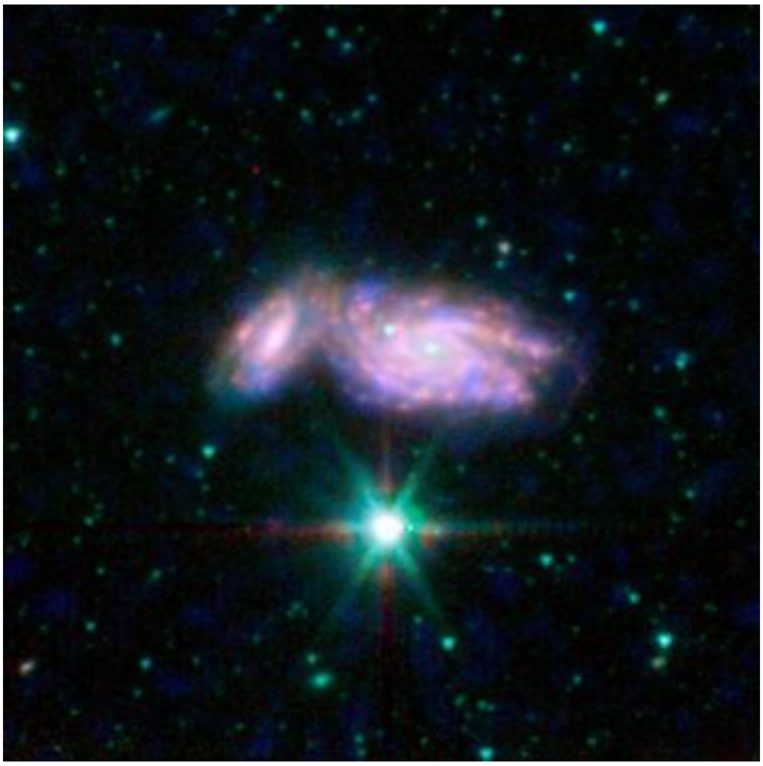


Figure 29 Viewing with UV light (Picture from NASA)

X-ray Telescopes

It is hard to reflect X-rays like light from an ordinary mirror; they are transmitted or absorbed. They can be reflected using a technique called glancing mirrors, with the angle between the X-rays and the mirror being less than 2 degrees (angle of incidence 88 °). The mirrors are made of ceramic or metal foil. Modern instruments can handle X-ray photons of energy up to 78 keV. These instruments have discovered X-ray pulsars and "bursters".

Gamma Ray Telescopes

These are carried up above the atmosphere either by balloons or satellites. The photons are not collected with mirrors but are detected using complex arrays of charged coupled devices which track the passage of a gamma ray photon. Data are transmitted digitally, and powerful software is used to analyse the images. Such an image is shown below (*Figure 30*).

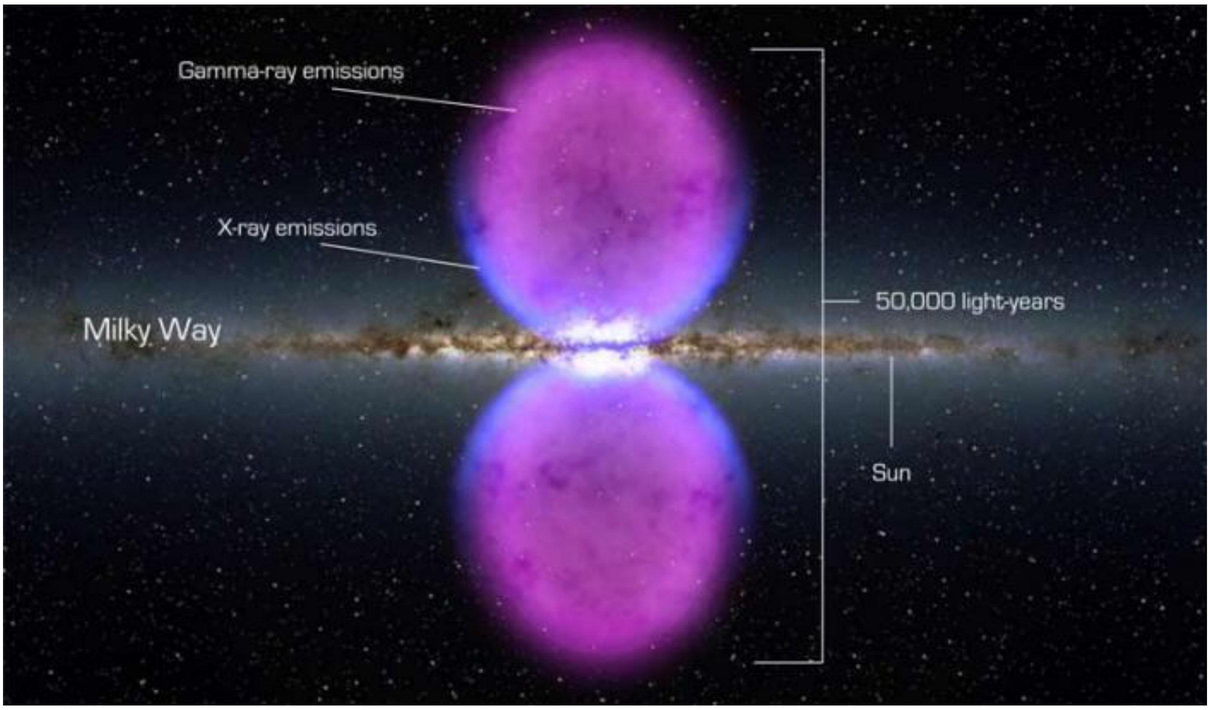


Figure 30 Observation using gamma rays (Photo from NASA)

The picture shows two vast gamma ray bubbles appearing symmetrically from a super-massive **black hole** that is thought to exist in the centre of the Milky Way galaxy. Gamma ray bursts are observed when very large supernovae (**hyper-novae**) collapse into black holes.

14A.035 Using Telescopes

The table below shows the uses and drawbacks of using different kinds of instruments:

Type	*Location*	*Wavelength*	*Resolution (degrees)*	*Advantages*	*Disadvantages*
Radio	Ground	1 mm - 10 m	0.2	Radio waves pass through dust and the atmosphere	Steerable dish is large, complex, and expensive.
IR	Ground or in space	700 nm – 1 mm	5.7×10^{-5}	Can detect warm but invisible objects in space.	Mirror has to be cooled
Light	Ground or in space	600 nm - 300 nm	8.5×10^{-6}	Detailed images possible, images are easily interpreted, telescopes can be accessed easily.	Ground instruments have images distorted by refraction and turbulence of the atmosphere.
UV	Space	300 nm – 10 nm	10^{-7}	Can detect very hot objects.	Has to be in space.
X-ray	Space	10 nm – 0.01 nm	10^{-7}	Detect X-ray pulsars	Has to be in space. Optics are complex.
Gamma	Space	10^{-11} m – 10^{-14} m	0.2	Detect γ ray burst from supernovae.	Has to be in space. γ rays are not possible to focus, so image is limited by the CCD array.

14A.036 Problems of Observing Stars

Meaningful astronomy is getting harder. Research astronomers are generally to be found in the major universities whose observatories were often set up in major cities. The observatories are often quite old. Although the instruments are very capable, their situation limits them:

- Light pollution from street lights makes observation of dim objects difficult.
- If the weather is cloudy, observation is impossible.
- Dust from pollutants can interfere with images.
- The atmosphere is turbulent, leading to scintillation.

These problems can be solved:

- by moving observatories to the top of high mountains.
- mounting a telescope in an aeroplane (although good tracking depends on the skill of the pilot and absence of turbulence that can make for a bumpy ride).
- Putting the telescope in orbit.

If other radiations are being investigated, satellites equipped with appropriate receivers are used.

Questions

Tutorial 14A.03

14A.03.1

How much more powerful is the radio telescope at Arecibo in Puerto Rico (page 29) than the one at Jodrell Bank?

14A.03.2

What disadvantage does the Puerto Rico instrument have over the one at Jodrell Bank?

14A.03.3

What is the angular resolution of 10 metre wavelength radio waves by the Jodrell Bank Telescope whose diameter is 75 m?

2. Observing Stars

Tutorial 14A.04 Stellar Distances and Magnitudes

AQA Syllabus

Contents

14A.041 Astronomical distances

Space is very big. The table gives the time for light to travel various distances. Light travels at 3.0×10^8 m s^{-1} = 300 000 km s^{-1}.

Journey	*Time*
Sun to Earth	500 s
Sun to Jupiter	40 min
Sun to Pluto	6 hours
Sun to Proxima Centauri (nearest star)	4 years
Across the Milky Way	100 000 years
Galaxy to Galaxy	1 000 000 years

Kilometres are far too small a measurement to be useful. Measuring space distances is called **astrometry**.

14A.042 The Parsec

Astronomers use the idea of **parallax** to estimate the distances of stars. You will know about parallax as a source of error in reading instruments. If your eye is not in the right place, your reading will be offset (either too high or too low) as your eye is not in line with the pointer. On a train, you will see a tree in a nearby field moving relatively quickly compared to the hills in the distance. This is due to parallax (*Figures 31* and *32*).

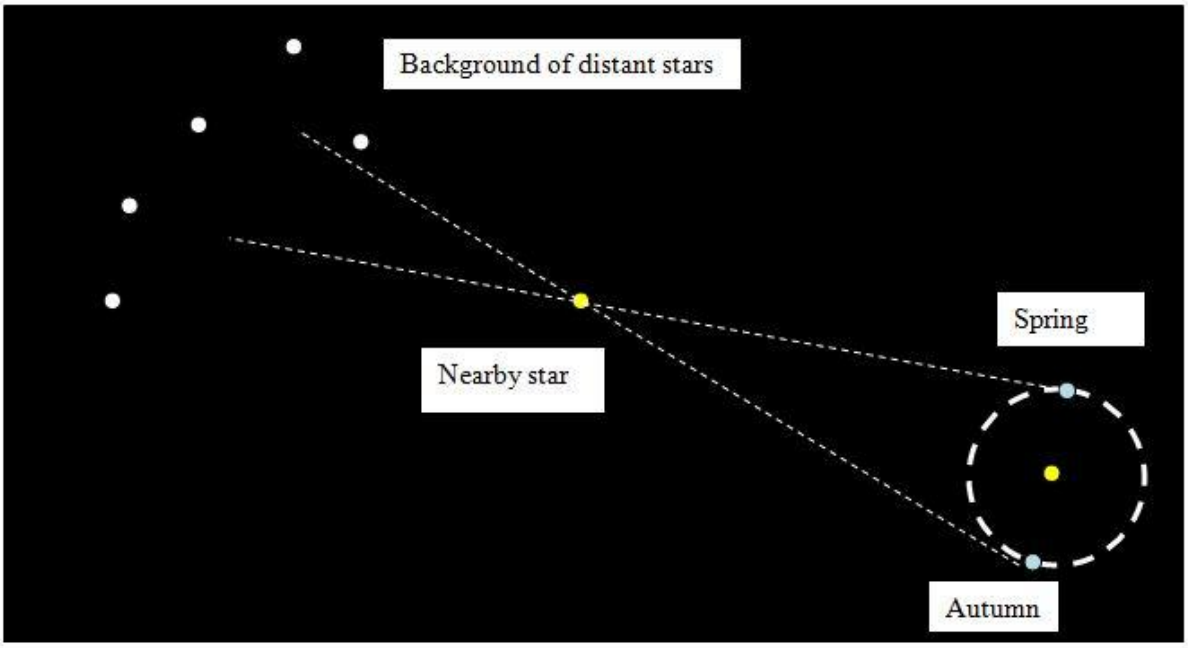
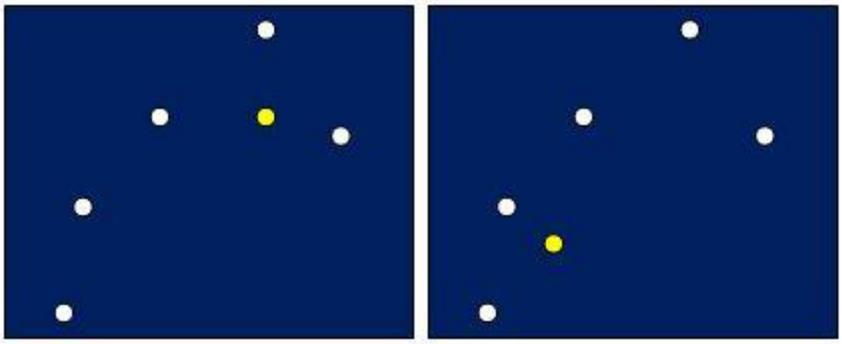


Figure 31 How parallax arises

Stars appear offset against the background of other stars, depending on the season:



Autumn Spring

Figure 32 Showing the effects of parallax

In the autumn, the following pattern is observed. The background of distant stars remains the same, but the nearby star appears to have moved to the right. In the spring, the background pattern is observed as before. However, the nearby star appears to have moved to the left.

In the winter you might not see the star as it will be behind the Sun.

In Summer, the nearby star will be halfway between the two positions.

The closer the star, the more rapidly it will appear to move, and the variations in position will be greater.

To work out **the parallax angle**, we can draw a diagram like this (*Figure 33*).

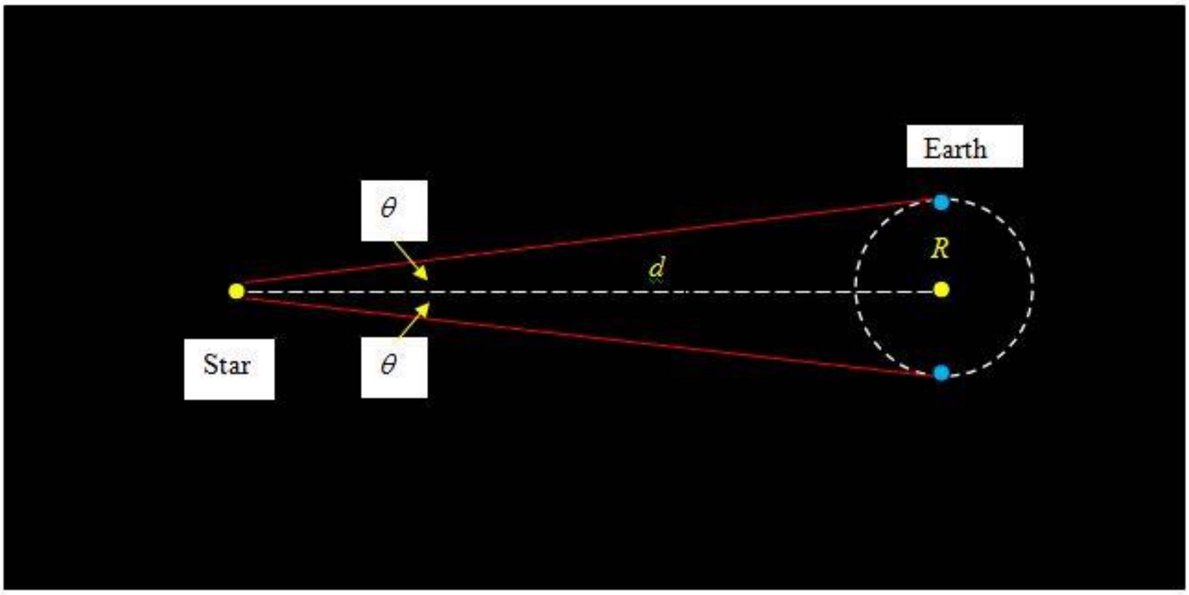


Figure 33 Parallax angle

Let the distance from the Sun to the star be d. Let the radius of the Earth's orbit be R. The angle can be worked out using the **tangent** function:

$$\tan\theta = \frac{R}{d}$$

.... *Equation 16*

Since $\theta \approx \sin\theta \approx \tan\theta$ for small angles in radians, we can write:

$$\theta = \frac{R}{d}$$

.... *Equation 17*

The distance d is very much bigger than R, so the angle is going to be very small.

Although we have stressed the **radian** as the units to measure angles in, astronomers often use **degrees**. Angles of less than 1 degree are often measured in **minutes**, where 1 minute is 1/60 degree. An even smaller angle is used called an **arc second**, where 1 arc second = 1/60 minute = 1/3600 degree. This is the basis of the **parsec** (pc) which is defined as:

the distance to a star that subtends an angle of one arc second to the line from the Earth to the Sun

The diagram shows the idea (*Figure 34*).

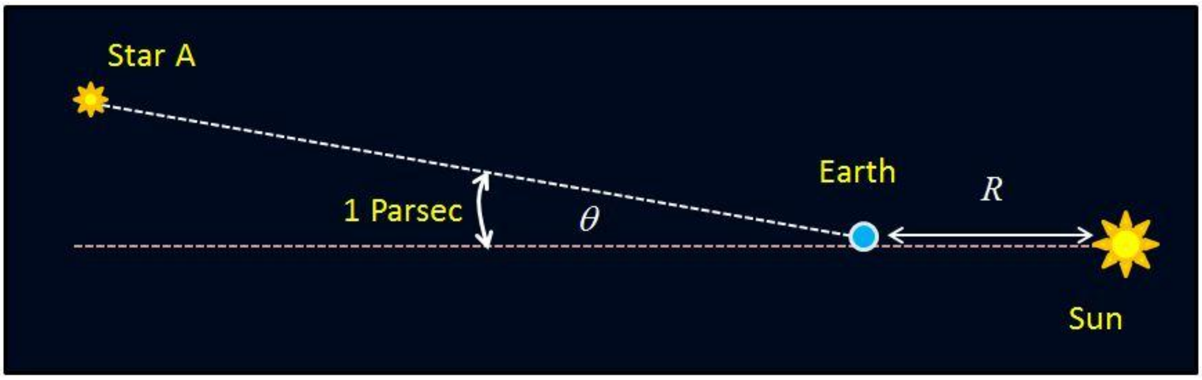


Figure 34 The Parsec

On this diagram, the distance, R, is the distance from the Earth to the Sun in **astronomical units**.

1 astronomical unit = 1.496 × 10^{11} m

Now 1 degree = 2π ÷ 360 = 0.0175 rad

1 arc second = 0.0175 ÷ 3600 = 4.85 × 10^{-6} rad.

Since

$$d = R/\theta \text{ .... Equation 18}$$

$$\boldsymbol{d = 1.496 \times 10^{11}\ \text{m} \div 4.85 \times 10^{-6}\ \text{rad} = 3.09 \times 10^{16}\ \text{m}}$$

1 light year = 3.00 × 10^{8} m s^{-1} × 365 d × 86400 s d^{-1} = 9.46 x 10^{15} m

Therefore

$$d = 3.09 \times 10^{16}\ \text{m} \div 9.46 \times 10^{15}\ \text{m ly}^{-1} = \mathbf{3.26\ light\ years}.$$

If we measure the angle of parallax, we can measure the distance from the Sun by the equation:

$$d = \frac{1}{\theta}$$

.... *Equation 19*

The figure 1 refers to the distance from the Sun in astronomical units. The term *d* is the distance in parsecs, and θ is the angle in **arc-seconds**. The smaller the angle in arc seconds, the further the star.

Before using this equation, make sure that the angle is in parsec, NOT radians.

The table shows the units used in astronomy and astrophysics.

Unit	***What it is***	***Abbreviation***	***Conversion***
Astronomical Unit	Distance between Earth and Sun	AU	1 AU = 1.496×10^{11} m
Light Year	Distance travelled by light in 1 year	ly	1 ly = 9.46×10^{15} m
Parsec	An object at 1 pc subtends an angle of 1 arc second for a distance of 1 AU	pc	1pc = 3.086×10^{16} m 1 pc = 3.26 ly

Parallax methods work well up to about **100 pc** (which is quite a long way). Beyond that, the measurement of the parallax angle is not at all easy.

14A.043 Apparent Magnitude

The Greek astronomer **Hipparchus** (190 BC - 120 BC) classified stars according to their apparent brightness to the naked eye, about two thousand years ago. (In some texts you will see his name written as Hipparchos.) Its scale was 1 to 6. The scale is still used today and is called the **apparent magnitude scale**. The apparent magnitude is given the code m. Magnitude 1 stars are about 100 times brighter than magnitude 6 stars. A change in 1 magnitude is a change of 2.512 ($100^{1/5} = 2.512$). The scale is logarithmic because each step corresponds to multiplying by a constant factor.

The **apparent magnitude** is the brightness of a star as it **appears to the observer**.

To work out the apparent magnitude we need to:

- calculate the difference in apparent magnitudes= n.
- work out the ratio of the brightnesses = 2.512^n.

Worked example The Sun has a magnitude of -26.7. The moon has a magnitude of - 12.7. What is the ratio of their brightnesses?
Answer Difference in magnitudes = dimmest - brightest = -12.7 - -26.7 = +14 Ratio of brightnesses = 2.512^{14} = **398 000**

Therefore, the Sun is about 400 000 times brighter than the Moon. Notice that bright objects have negative magnitudes. The scale looks like this (*Figure 35*):

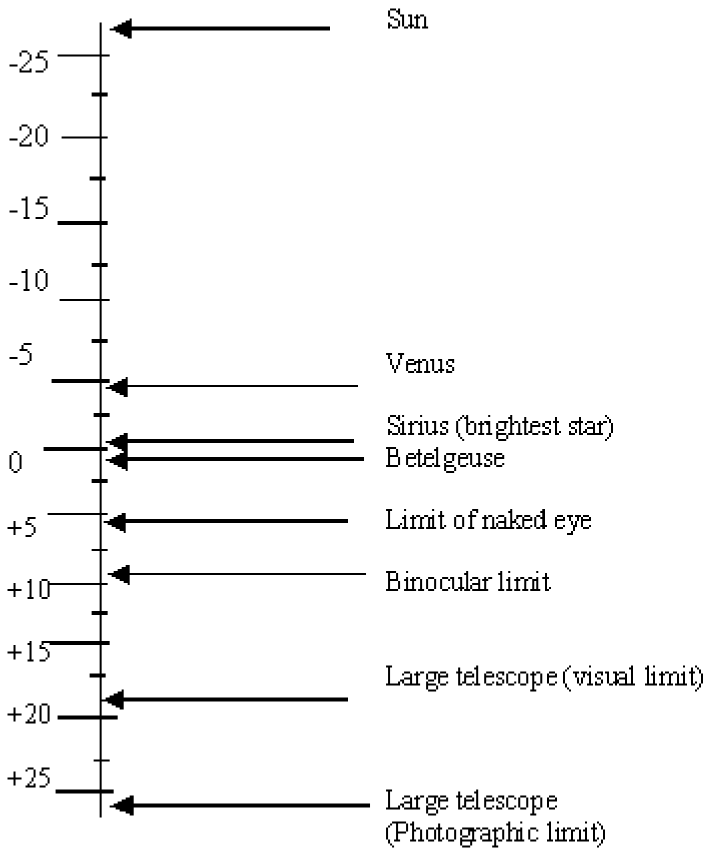


Figure 35 Apparent magnitude scale

14A.044 An Equation

Consider two stars that have apparent magnitudes m_A and m_B. They have intensities I_A and I_B respectively. The difference in magnitude is:

$$\Delta m = m_A - m_B$$ .... *Equation 20*

Each difference in magnitude of 5 gives an intensity increase of 100 times.

$$\frac{I_{\mathrm{A}}}{I_{\mathrm{B}}} = 100^{\frac{\Delta m}{5}}$$

.... *Equation 21*

This can be rewritten as:

$$\frac{I_A}{I_B} = 100^{0.2\Delta m}$$

.... *Equation 22*

Take logarithms to get rid of the powers:

$$\lg\left(\frac{I_A}{I_B}\right) = \lg 100 \times 0.2\Delta m = 0.4\Delta m$$

.... *Equation 23*

This rearranges to:

$$\Delta m = 2.5 \lg\left(\frac{I_A}{I_B}\right)$$

.... *Equation 24*

Maths Note

The term '$\lg$' stands for $\log_{10}$. Do not use natural logs here.

14A.045 Absolute Magnitude

In Topic 12, we saw that the **intensity** of gamma radiation reduced as an **inverse square law**. This is true for any radiation, including visible light. The relationship is:

$$I = \frac{k(I_0)}{x^2}$$

.... *Equation 25*

[I – intensity; I_0 – intensity at the source; k – constant; x – the distance from the source]

When doing intensity calculations, it is more common that we do not know what I_0 is. Instead, we have a count I_1 at point 1 and a count I_2 at point 2 at distances x_1 and x_2 respectively.

So, we can write:

$$I_1 = \frac{k(I_0)}{x_1^2}$$

.... *Equation 26*

and

$$I_2 = \frac{k(I_0)}{x_2^2}$$

.... *Equation 27*

We can combine *Equations 26* and *27* in a rearranged form to give us:

$$I_1(x_1)^2 = kI_0 = I_2(x_2)^2$$ ………….. *Equation 28*

So, we can write:

$$I_1 x_1^2 = I_2 x_2^2$$

.... *Equation 29*

If we rearrange further, we can get a ratio:

$$\frac{I_1}{I_2} = \left(\frac{x_2}{x_1}\right)^2$$

.... *Equation 30*

The apparent magnitude represents the **intensity**, i.e. the power per square metre.

Consider two stars A and B that have intensities I_A and I_B respectively. Suppose the stars had apparent magnitudes m_A and m_B. The difference in magnitude is:

$$\Delta m = m_A - m_B$$ ……….... *Equation 31*

Now, every difference of 5 magnitudes gives an increase in intensity of 100 times. We can give a general rule as:

$$\frac{I_A}{I_B} = 100^{\frac{\Delta m}{5}}$$

.... *Equation 32*

We can rewrite this as:

$$\frac{I_A}{I_B} = 100^{0.2\Delta m}$$

.... *Equation 33*

Each increase in magnitude does NOT give an increase in intensity of 20 times.

We can get rid of the power by taking logarithms. This time we use $\log_{10}$ (lg) not ln. lg (100) = 2. If one number is divided by a second number, the log of the second number is subtracted.

Therefore, we can write *Equation 33* as:

$$\lg(I_A) - \lg(I_B) = 2 \times 0.2\ \Delta m = 0.4\ \Delta m$$ .... *Equation 34*

Rearranging *Equation 34* in terms of Δm:

$$\Delta m = 2.5 \lg\left(\frac{I_A}{I_B}\right)$$

.... *Equation 35*

Since:

$$\frac{I_1}{I_2} = \left(\frac{x_2}{x_1}\right)^2$$

.... *Equation 36*

we can write:

$$\Delta m = 2.5 \lg\left(\frac{d_B}{d_A}\right)^2$$

.... *Equation 37*

where d is the distance of the star. The log of a number squared is twice the log of the number, so:

$$\Delta m = 5 \lg\left(\frac{d_B}{d_A}\right)$$

.... *Equation 38*

When we use **absolute magnitude**, we "place" all stars at an arbitrary distance of **10 parsec**. We give the absolute magnitude the code M. So, we are comparing the brightnesses of all stars as if they were at 10 pc.

So, let's think of another star at a distance d which has an apparent magnitude of m. We can work out the absolute magnitude by moving the star to 10 parsec (in our minds, of course):

$$\Delta m = 5\lg\left(\frac{d}{10}\right)$$

...............*Equation 39*

We can rewrite this as:

$$m - M = 5\lg\left(\frac{d}{10}\right)$$

.... *Equation 40*

The absolute and apparent brightnesses of stars are related by this formula:

$$m - M = 5\lg\left(\frac{d}{10}\right)$$

.... *Equation 41*

[m - apparent magnitude; M - absolute magnitude; d - distance (pc)]

Make sure that you use $\log_{10}$ not $\log_e$.

Worked Example
The apparent magnitude of Sirius = -1.46. It is at a distance of 4 light years (ly) from Earth. What is its absolute magnitude?
Answer Convert ly to pc: 4 ÷ 3.26 = 1.23 pc $-1.46 - M = 5 \log 1.23/10$ $-M = (5 \times -0.910) + 1.46$ $-M = -4.55 + 1.46 = -3.09$ $M =$ **+3.09**

If the star is less than 10 pc away, then the absolute magnitude tends to move in the positive sense, which means that they become dimmer. Stands to reason. If you move a star further away, the dimmer it gets. Stars over 10 pc will have absolute magnitudes that are brighter than their apparent magnitude.

The answer to Question 14A.04.4 should tell you that the Sun viewed from 10 pc would glow with a brightness similar to many other stars.

Questions

Tutorial 14A.04

14A.04.1

What is 10 parsec in kilometres? A supersonic plane is travelling at 3000 km h^{-1}. How long would it take to travel 10 parsec?

14A.04.2

How much brighter is Sirius (m = -1.46) than Betelgeuse (m = +0.50)?

14A.04.3

What do you think is a problem with the apparent magnitude scale?

14A.04.4

The apparent magnitude of the Sun is -26.7 What is its absolute magnitude? How does it compare with the absolute magnitude for the star Alpha Centauri which is +4.38?

14A.04.5

Bellatrix and Elinath are two stars with the same apparent magnitude. The distance from the

Earth to Bellatrix is 470 light years and its absolute magnitude is -4.2.

(i) Calculate the distance to Bellatrix in parsecs.

(ii) Calculate the apparent magnitude of Bellatrix.

(iii) Elinath has an absolute magnitude of -3.2. State, giving a reason, which of the two stars is closer to the Earth.

(AQA Past Question)

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14A.051 Star Light

Stars at night appear to be white. This happens because the light intensity through our unaided eye is so low that the cells in our eyes that pick up colour are not activated. So, we see in black and white. With good telescopes we can see the different colours of the stars. We can capture the colours using charged coupled devices or sensitive colour chemical film.

Stars glow in the same way as other glowing objects. If we turn the voltage up across a light bulb from zero volts up to its normal voltage, we see the filament glow a dull red, then to orange, to yellow to white. If we look at a spectrum as this happens, we see that:

- there is a continuous range of colours,
- but the relative intensity changes.

See *Figure 36*.

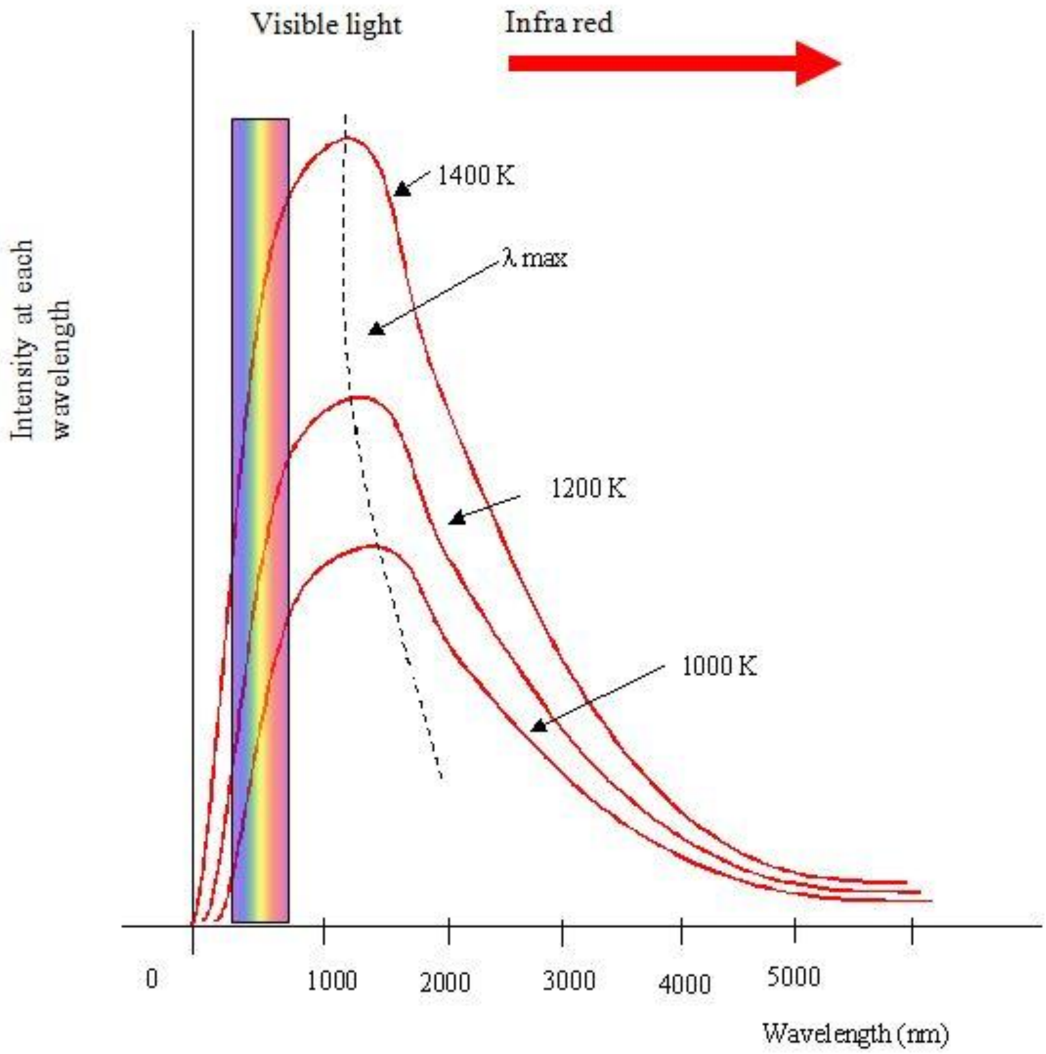


Figure 36 Plot of intensity against wavelength

The light that we see is the resultant of that mixture of colours and other wavelengths. On this graph, the visible spectrum is to the left, between 300 and 600 nm. To the right are wavelengths of infra-red radiation.

We do not see green stars because even if the peak wavelength were in the green region, 500 nm, there are also red and blue components as well. Therefore, the star appears white, because red, green, and blue make white.

14A.052 Black Body Radiation

We look at the temperature of stars by looking at their colours. A lot of energy is given off as **thermal radiation.** Objects that are red hot have a temperature of about 1200 K. To understand how the colour of an object depends on its temperature, we need to understand the concept of a **black body**. A black body is a perfect absorber so that all radiation that falls on it is absorbed.

A black body is NOT the same as a black hole.

A perfect absorber is a perfect emitter. Therefore, if we heat it up it will emit radiation including visible light. This is true (to a first approximation) for stars. Note the following for black bodies:

- a hot object emits radiation across a wide range of wavelengths.
- there is a peak in intensity at a given wavelength.
- the hotter the object the higher the peak.
- the hotter the object the shorter the peak wavelength.
- the **area under the graph** is the total energy radiated per unit time per unit surface area.

This is shown in the graph (*Figure 37*).

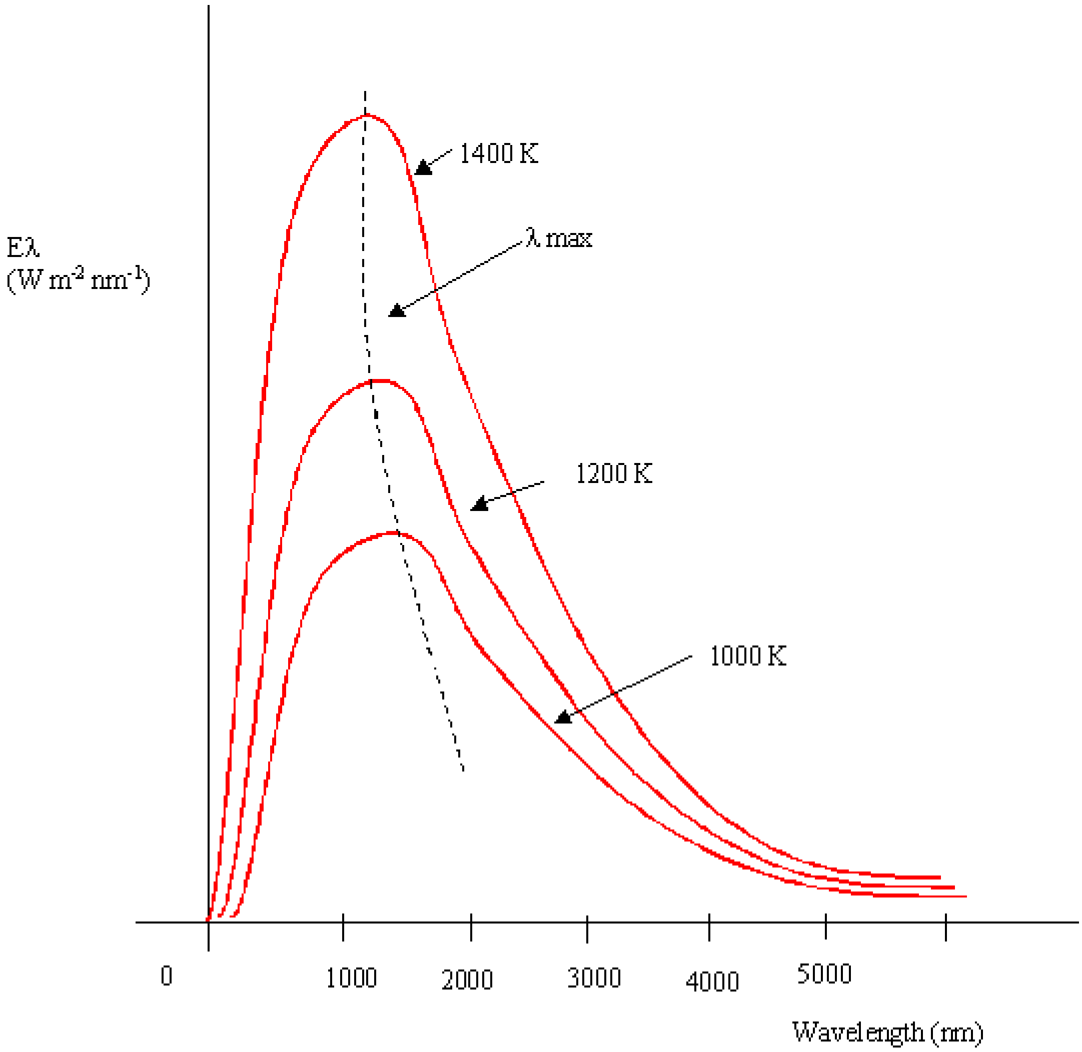


Figure 37 graph showing the wavelength at which maximum energy is radiated at different temperature.

The **peak wavelength** is λ_{max} which is the **wavelength at which maximum energy is radiated**. This is **inversely proportional** to the **Kelvin** temperature. It is called **Wien's Displacement Law** (as the peak is displaced towards shorter wavelengths). We write it as:

$$\lambda_{max}\, T = \textbf{constant} = \textbf{0.00289 m K}$$

Worked example What is the peak wavelength of a black body emitting radiation at 2000 K? In what part of the electromagnetic spectrum does this lie?
Answer λ_{max} = 0.00289 m K ÷ 2000 K λ_{max} = **1.45×10^{-6} m** = 1450 nm This is in the infra-red region.

You don't get green stars because the light from stars is emitted at a range of wavelengths, so there is mixing of colours. So those stars with a λ_{max} in the green region will actually appear to be white.

14A.053 Classification by Temperature

The American astronomer Anne Cannon was the first to classify stars using the science of **spectroscopy**. Astronomers look for **Balmer lines** which arise from electron transitions in hydrogen atoms (See Topic 3). As the electron drops from high levels to the **second energy** level (the one above the ground state), photons are emitted. Because they are emitted in random directions, against a complete spectrum of colours the emissions of photons would appear **black**. This is an **absorption spectrum** (*Figure 39*). Compare the **emission spectrum** (*Figure 38*)

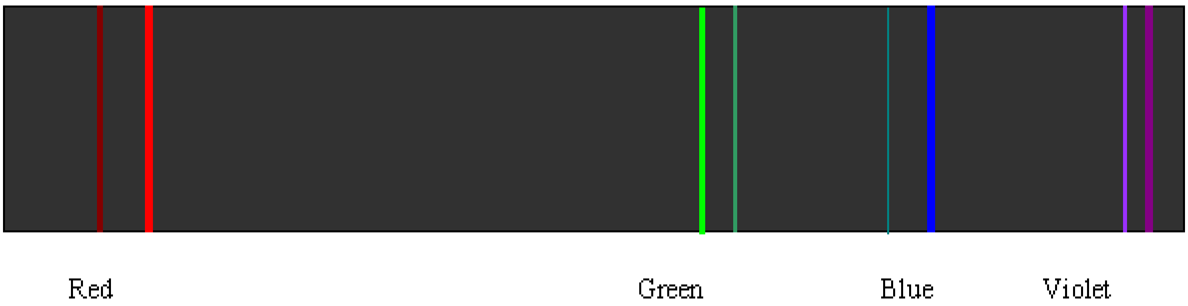


Figure 38 Emission spectrum

with the **absorption spectrum**:

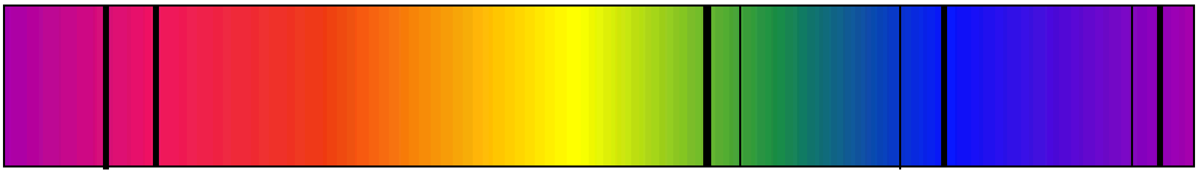


Figure 39 Absorption spectrum

By studying the **spectral absorption lines** at wavelengths corresponding to the photons of the **Balmer** series, astronomers can get an idea of the temperature:

- At low temperatures there are few violent collisions to excite electrons which remain in the ground state. Energy level changes are rare.
- At high temperatures, there are many violent collisions between atoms. Electron transitions occur at higher levels so there are comparatively few Balmer transitions.
- At intermediate levels many electrons are performing Balmer transitions, so there are strong absorption lines.

The Balmer lines are seen in the **corona** of the star. The **Balmer** lines (*Figure 40*) are the transitions in between energy levels in the hydrogen atom that end at the energy level $n = 2$ ($n = 1$ is the **ground state**).

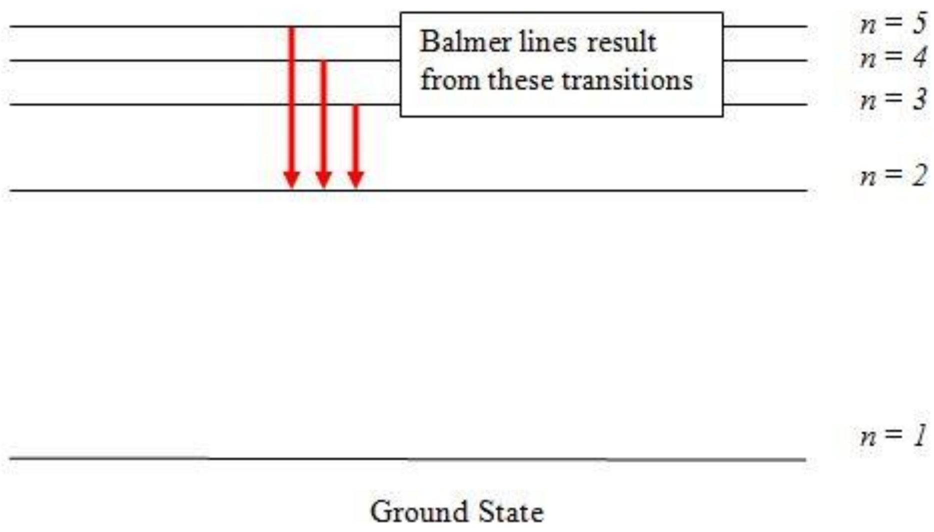


Figure 40 Balmer lines for a hydrogen atom

The temperatures can be related to the transitions. Each level has a definite energy level. Remember from Topic 3 that:

$$E_3 - E_2 = hf = \frac{hc}{\lambda}$$ .... *Equation 42*

So, we can work out the photon wavelength for the transition $n = 3$ to $n = 2$.

Question 14A.05.2 asks to work out the wavelength of a photon associated with the electron transition between levels $n = 2$ and $n = 3$. Now we have a wavelength, we can use the Wien equation to calculate the temperature:

$$\lambda_{max}\, T = 0.00289 \text{ m K}$$ …………… *Equation 43*

The graph of intensity against temperature looks like this (*Figure 41*).

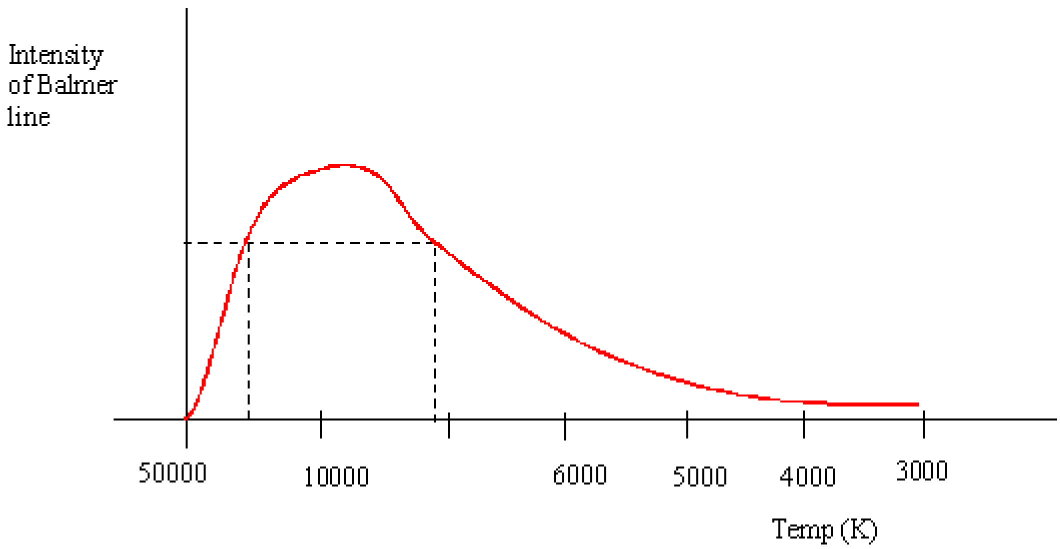


Figure 41 Balmer line intensity against Kelvin temperature

Notice that:

- temperature **decreases** from left to right.
- for a given intensity, two temperatures are possible.

To overcome this, the spectra of other elements are analysed. Peak intensities of different elements are found, and this can tie down the temperature. The idea is shown on the next graph (*Figure 42*).

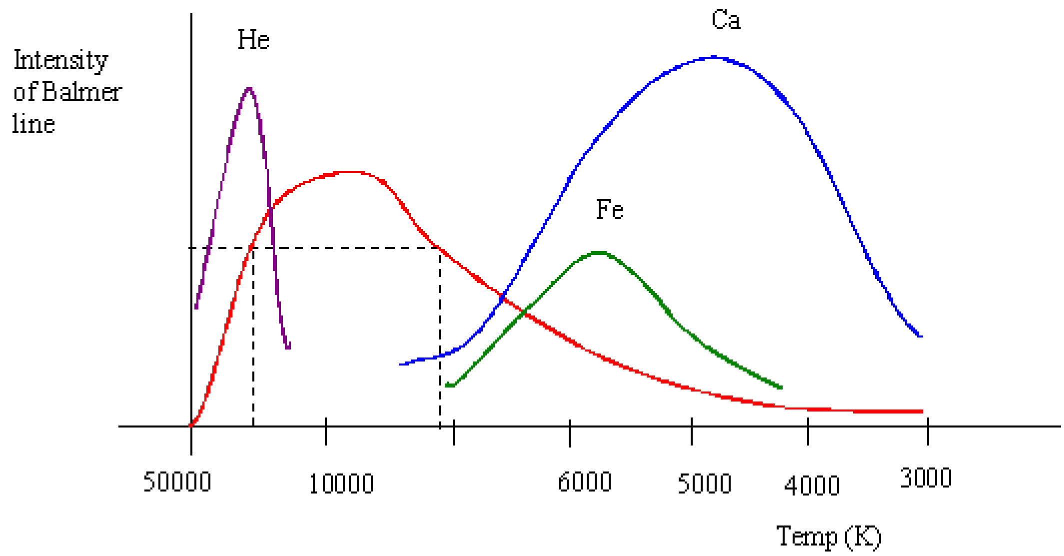


Figure 42 peak intensities for other elements in a star

Therefore, in the Sun, the spectral lines would be seen for iron and calcium, indicating a surface temperature of about 6000 K. Very hot stars show spectral lines for light elements while cool stars will show up heavy elements, and spectra for molecules as well.

14A.054 Spectral Classes of Stars

The table shows the spectral classes for stars:

Spectral Class	Surface temp (K)	H Balmer Series	Other elements
O	40 000	weak	ionised He
B	20 000	medium	He atoms
A	10 000	strong	weak ionised Ca
F	7500	medium	weak ionised Ca
G	5500	weak	medium ionised Ca
K	4500	weaker	strong ionised Ca
M	3000	very weak	strong TiO

The classifications of stars according to spectra are also subdivided further with numbers (e.g. A5) to allow the surface temperature to be determined within about 5 %.

14A.055 Luminosity of Stars

The area under the graph above is related to the rate at which a black body radiates energy. The **luminosity** of a star is the total energy given out per second, so it's the **power**. From the graph the luminosity increases rapidly with temperature, which gives rise to **Stefan's Law**. Formally this is stated as:

The total energy per unit time radiated by a black body is proportional to the fourth power of its absolute temperature.

In other words, double the temperature and the power goes up sixteen times. In physics code we write:

$$P = \sigma A T^4$$ .... *Equation 44*

[P- Power (W); σ - Stefan's constant; A - area (m^2); T - temperature (K)]

The strange looking symbol σ is "sigma", a Greek letter lower case 's'. It is **Stefan's Constant**.

σ = **5.67 x 10^{-8} W m^{-2} K^{-4}** .... *Equation 45*

We can treat a star as a perfect sphere ($A = 4\pi r^2$) and a perfect black body. So, for any star, radius r, we can write:

$$P = 4\pi r^2 \sigma T^4$$ .... *Equation 46*

(Note: in some text books the power may be represented as **luminosity** with the physics code L)

Stars with the same **absolute magnitude** have the same **power output**. We can justify this statement by considering stars P and Q:

- Power of P $= P_P = A_P \sigma T_P^4$ where A_P is the area of P and T_P is the surface temperature of P.
- Power of Q $= P_Q = A_Q \sigma T_Q^4$ where A_Q is the area of Q and T_Q is the surface temperature of Q.

We can equate the two expressions to give:

$$A_P \sigma T_P^4 = A_Q \sigma T_Q^4$$ .... *Equation 47*

So, we can write:

$$\frac{A_P}{A_Q} = \frac{T_Q^{\ 4}}{T_P^{\ 4}}$$ .... *Equation 48*

So, if the temperatures are the same, the areas will be the same. Therefore, the radii will be the same.

The intensity of the Sun's radiation decreases by an inverse square law. Therefore Saturn, about 10 times further from the Sun (i.e. 10 AU) receives only 1 % of the intensity of the Sun's radiation as Earth does.

14A.056 Inverse Square Law and Luminosity

As the energy leaves a star, it becomes more spread out. The intensity of the radiation is the **power per unit area**, and is sometimes called the **flux**, given the physics code F. The **luminosity** of the star is the power of the star, given the physics code L.

We are a distance *d* from a star of luminosity L. The radiation propagates radially as shown (*Figure 43*).

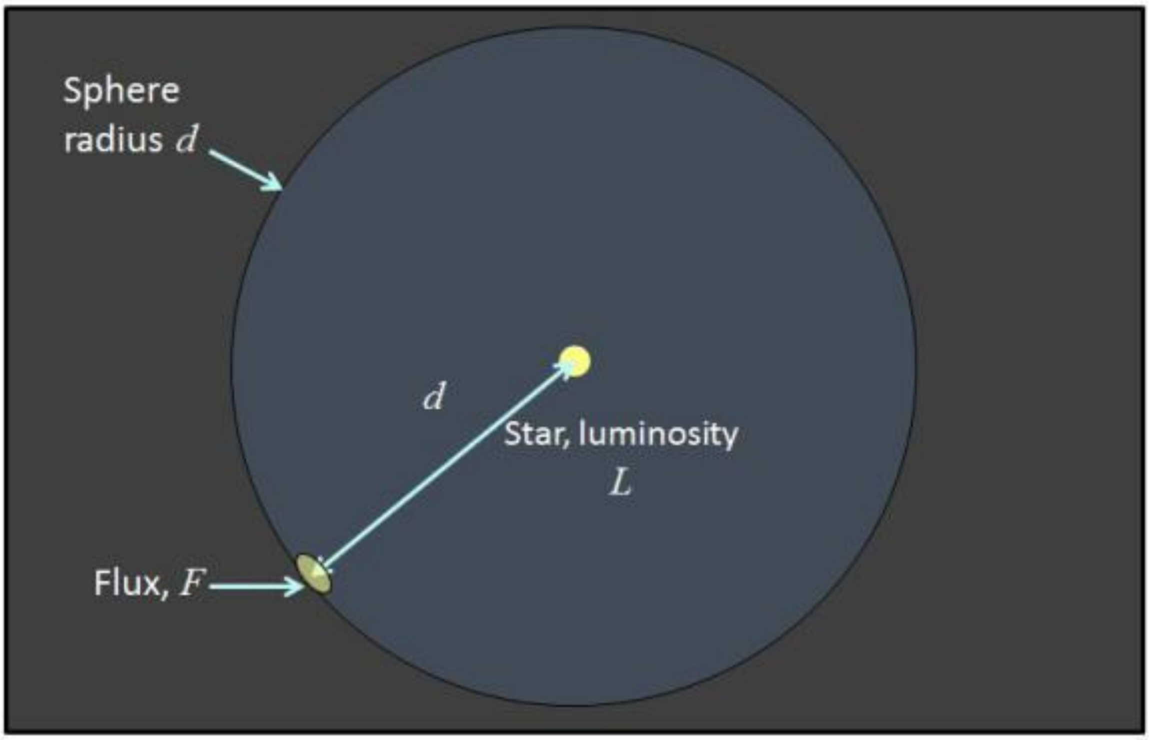


Figure 43 Flux F from a star of luminosity L

The total area of the sphere is given by:

$$A = 4\pi d^2 \quad \text{............ Equation 49}$$

The radiation flux is the radiation energy per unit area, so the flux is given by the equation:

$$F = \frac{L}{4\pi d^2} \quad \text{............. Equation 50}$$

Of course we can use the code r for the distance.

14A.057 The Power of the Sun

On the equator, the average intensity of the Sun's rays is about 1400 W m^{-2}. In practice, some is absorbed by the atmosphere, and some is reflected as heat, but we will use this in a calculation to work out the power given out by the Sun. See *Figure 44*.

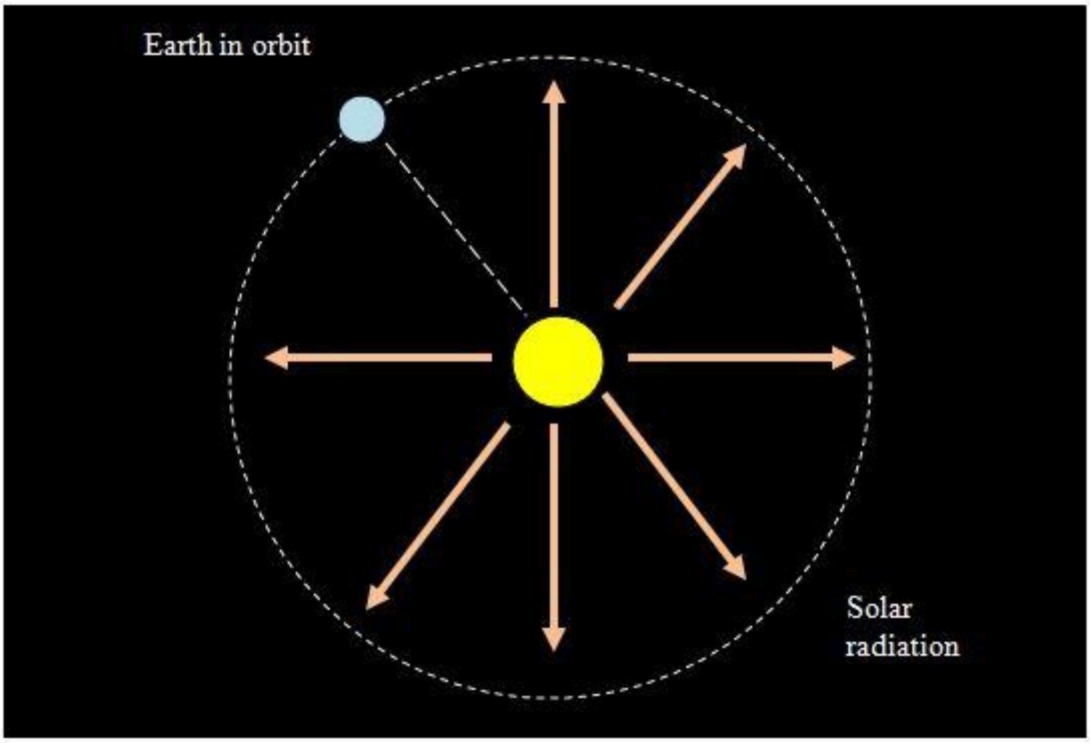


Figure 44 Radiation from the Sun

We can work out the power of the Sun by working out the total area of a sphere that has the radius of the Earth's orbit.

$$A = 4\pi r^2 = 4 \times \pi \times (1.50 \times 10^{11})^2 = \underline{\mathbf{2.83 \times 10^{23}\ m^2}}$$

Since each square metre receives 1400 W, the total power of the Sun is:

$$2.83 \times 10^{23}\ \text{m}^2 \times 1400\ \text{W m}^{-2} = \underline{\mathbf{3.96 \times 10^{26}\ W}} = 4.0 \times 10^{26}\ \text{W (to 2 s.f.)}$$

You may have noticed that this figure is slightly lower than the answer worked out in question 14A.05.6, but some energy is absorbed and reflected, so 1400 W m^{-2} is slightly too low.

The Sun has an absolute magnitude of +4.8.

Giant stars and Dwarf Stars

It is not possible to measure the diameter of a star directly, but we can use the absolute magnitude of the Sun (+4.8) and the other star to get an idea of the distance. Consider the fictional Star PLC A2.47 which has an absolute magnitude of -0.2.

From your answer to question 14A.05.7 you should see that the power of PLC A2.47 is therefore 4.0×10^{28} W.

We can use Stefan's Law to work out the diameter of our star and compare it to the Sun. We use

$$P = A\sigma T^4$$ .... *Equation 51*

Rearrange the equation for σ:

$$\sigma = \frac{P}{AT^4}$$ .... *Equation 52*

We can equate the expressions for the Sun (denoted by the subscript P in *Equation 53*) and PLC A2.47 (denoted by the subscript Q):

$$\frac{P_P}{A_P {T_P}^4} = \frac{P_S}{A_S {T_S}^4}$$ .... *Equation 53*

A **dwarf** star is one that is much smaller than the Sun.

A **giant** star is one that is much bigger than the Sun. PLC A2.47 is a giant star.

Stefan's Law applies across the whole spectrum while magnitudes relate to the visible spectrum only. Some stars release their radiation in wavelengths that are NOT in the visible region. The magnitudes of such stars have to be modified to take this into account, which is not on the syllabus.

Questions

Tutorial 14A.05

14A.05.1

Betelgeuse appears to be red. If red light has a wavelength of about 600 nm, what would the surface temperature be?

14A.05.2

Calculate the photon wavelength for the transition $n = 3$ (-1.51 eV) to $n = 2$ (-3.41 eV) for the hydrogen atom.

What colour is this light?

14A.05.3

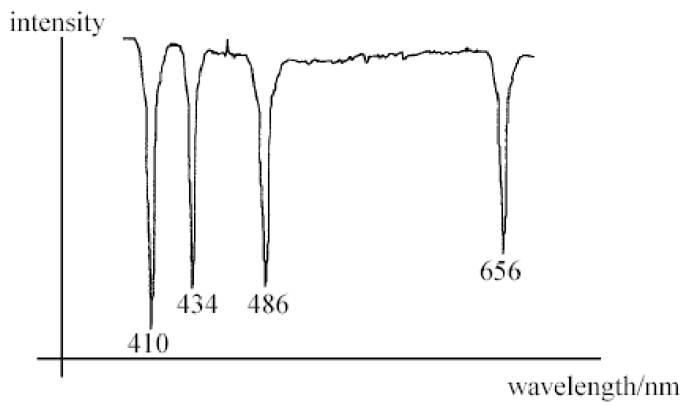
Use your answer to Question 2 to calculate the temperature associated with this wavelength.

14A.05.4

What would you not see when looking at the spectrum of the red giant Betelgeuse? What elements would you expect to see?

14A.05.5

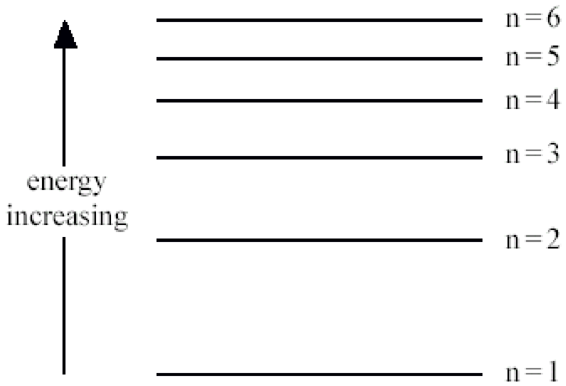
The graph shows part of the visible spectrum for the star Vega: (note that it is an absorption spectrum, so the intensity dips to a minimum at the emitted wavelengths.)



The absorption lines are due to excited hydrogen atoms. The wavelength of each absorption is given in nm.

(a) Explain how Hydrogen atoms produce these absorption lines.

(b) The diagram below shows the first six energy levels of a hydrogen atom. State which is the largest energy transition which produces an absorption line in the visible spectrum of Vega.



(c) State the value of the wavelength corresponding to this transition.

(d) What is the name given to the series which gives rise to the visible region of the hydrogen

spectrum?

(e) For which spectral classes are these lines the dominant feature?

(AQA Past Question)

14A.05.6

If the Sun has a radius of 6.96×10^8 m and a surface temperature of about 6000 K, what is its total power output? What is the power per unit area? What is the peak wavelength?

14A.05.7

Star PLC A2.47 which has an absolute magnitude of -0.2.

Show that PLC A2.47 has a power that is 100 times greater than the Sun.

14A.05.8

The Sun has a diameter of 1.4×10^9 m and a surface temperature of 5800 K

(a) Calculate the area of the Sun.

(b) Star PLC A2.47 is a G class star with a surface temperature of 5500 K. Calculate the diameter of PLC A2.47.

(c) Compare the size of PLC A2.47 with the Sun.

Tutorial 14A.06 Life and Death of a Star

AQA Syllabus

Contents

14A.061 Evolution of Stars

We can classify people by all sorts of different ways such as sex, race, creed, political beliefs and so on. In the last topic we saw how we can classify stars according to their **apparent** and **absolute magnitude**, and their **temperature** and **spectral analysis**. However, these classifications do not tell us a great deal about the age of the star or how it has **evolved**.

A Danish astronomer Ejnar Hertzsprung recognised patterns within stars. Independently an American, H N Russell, came up with the same sort of idea which gave rise to a useful tool called the **Hertzsprung-Russell Diagram**. This is essentially a graph of temperature on the horizontal axis while on the vertical axis we can put the **absolute magnitude** or the **luminosity** compared with the Sun. Sun = 1.

A typical HR diagram is shown in *Figure 45*.

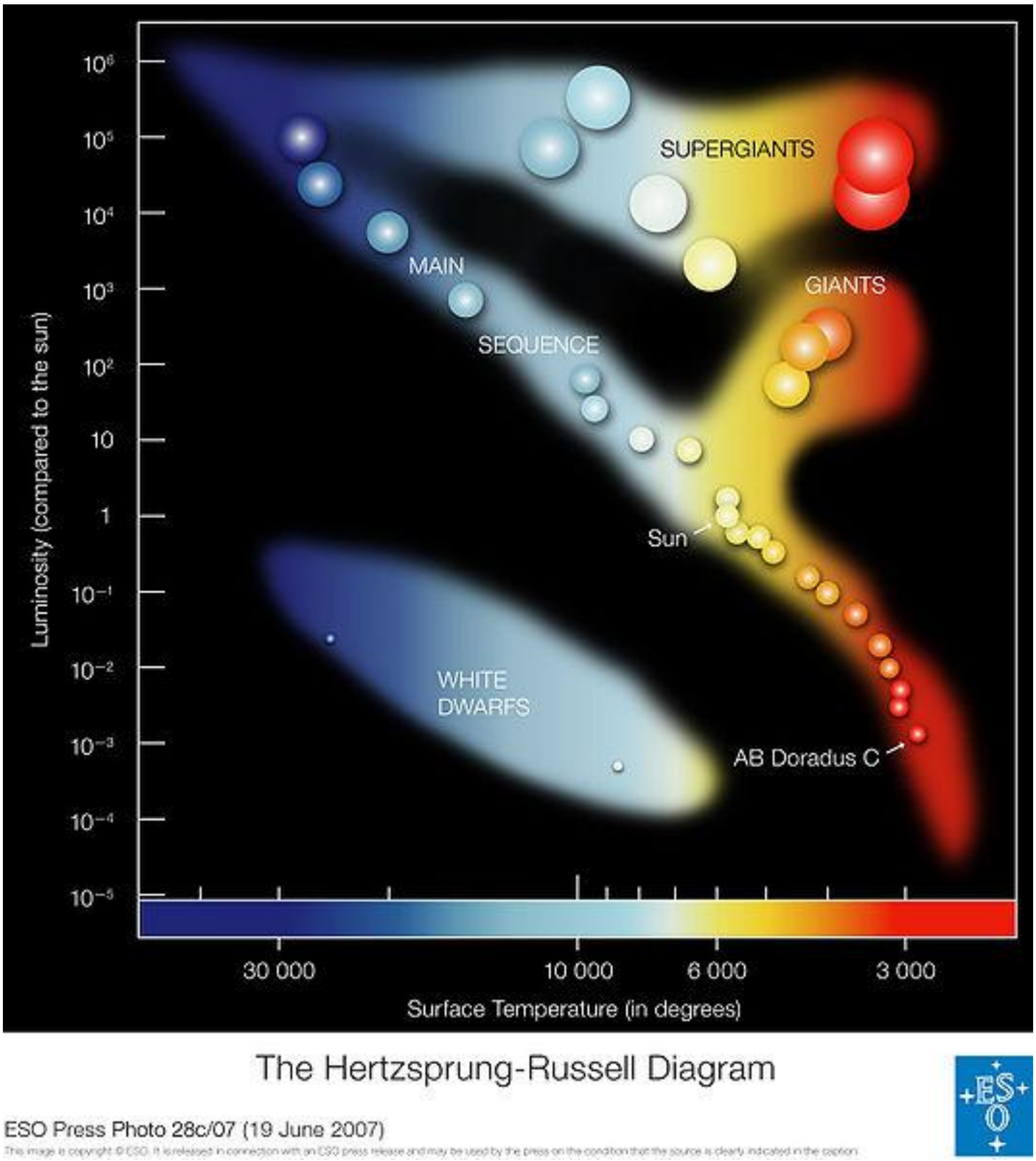


Figure 45 A Hertzsprung-Russell Diagram (European Southern Observatory, Wikimedia Commons)

Notice that

- the temperature scale is **decreasing**.
- the classes of star are placed alongside the temperature scale.
- the luminosity scale is **logarithmic** to compress it.

Most stars lie along the **main sequence**, going from very bright blue stars to very dim red stars. The Sun is somewhere in the middle of the main sequence. They range from cool low power stars of absolute magnitude +15 to very hot high power stars of absolute magnitude -5. The greater the mass of the star, the higher up the sequence it is. Mass ranges from 0.1 to 30 solar masses.

To the top right there two distinct classes of star, the **red giants** and the **red supergiants**. Although they are cool, they have to be big to achieve the luminosity. The star Betelgeuse would engulf the orbit of Jupiter. The magnitudes of these stars are between +2 and -2. They are 10 to 100 times larger than the Sun. They are more powerful than the Sun but have lower surfaces temperatures.

To the bottom left we have **dim** stars. Spectral line analysis suggests that they are very hot, but their low luminosity suggests that the stars are very small. **White Dwarfs** are thought to be about the size of the Earth but with a mass similar to the Sun. They have absolute magnitudes of +15 and +10. They are common but hard to observe.

Worked Example A red giant and a main sequence star both have an absolute magnitude of 0. Their surface temperatures are 4000 K and 10 000 K respectively. How any times larger is the red giant than the main sequence star?
Both stars have the same power output. Use the Stefan's Law: $$P = 4\pi r^2 s T^4$$ Let's call the red giant R and the mainstream M. We can fiddle with the formula to write: $$\frac{A_R}{A_M} = \frac{{T_M}^4}{{T_R}^4}$$ We can then put the numbers in: $$\frac{A_R}{A_M} = \frac{{T_M}^4}{{T_R}^4} = \frac{(10000\text{ K})^4}{(4000\text{ K})^4} = 39$$ Diameter of the red giant is the √39 = **<u>6.25</u>** times bigger.

14A.062 A Star is Born

Space is not a complete vacuum. There are about 10 atoms per cubic centimetre (compared to 10^{19} in a room). As well as **atoms** there are **molecules** and specks of **dust**. Many of these atoms and molecules come from stars that have exploded, which we will look at later.

Gravity is the driving force behind the birth of a new star. From your studies in Module 4 you will remember that gravity is a very weak force but has an infinite range. Gravity is always **attractive**, never repulsive. It pulls the particles together, and they accelerate inwards. The process is <u>very</u> slow indeed, but there is all the time in the universe for it to happen. The picture below shows such as dust cloud (*Figure 46*).

Figure 46 Dust clouds in space (Photo NASA)

As the particles come together, they collide increasingly frequently, and the temperature begins to rise. **Star formation** tends to happen where the clouds are dense and have a mass about a hundred times that of the Sun. There needs to be regions of non-uniform density. Stars are usually born in **clusters**.

As the gas cloud collapses and heats up, it will emit significant amounts of infra-red radiation. This is known as a **protostar**. At this stage the temperature is still too low for nuclear fusion to happen. If the mass is too low, the failed star ends up as a **brown dwarf**. Some astronomers consider Jupiter to be a failed star.

As the material gets hotter, molecules are torn apart and atoms are ionised. As the mixture gets hotter still a **plasma** is formed where atoms are stripped of most, if not all their electrons. Finally, an **ignition temperature** is reached and fusion starts. The temperature is about 15 million °C. The picture shows the glow of very young stars (*Figure 47*).

Figure 47 Young star are born (Photo NASA)

14A.063 Stable Phase of a Star

Nuclear fusion releases a lot of energy. Humans have achieved it but only in the context of an explosion that would make a thunderclap like a whisper. The largest fusion bombs used deuterium (an isotope of hydrogen) to produce Helium. The amount of fused gas used would fill little more than a large party balloon. So why does a star not fly apart?

There are two opposing forces:

- **gravity** trying to make the star collapse in on itself.
- the **outward force** of the explosion (sometimes called the **radiation pressure**).

In the life time of a star, the two forces balance each other out (*Figure 48*) and the star remains the same size.

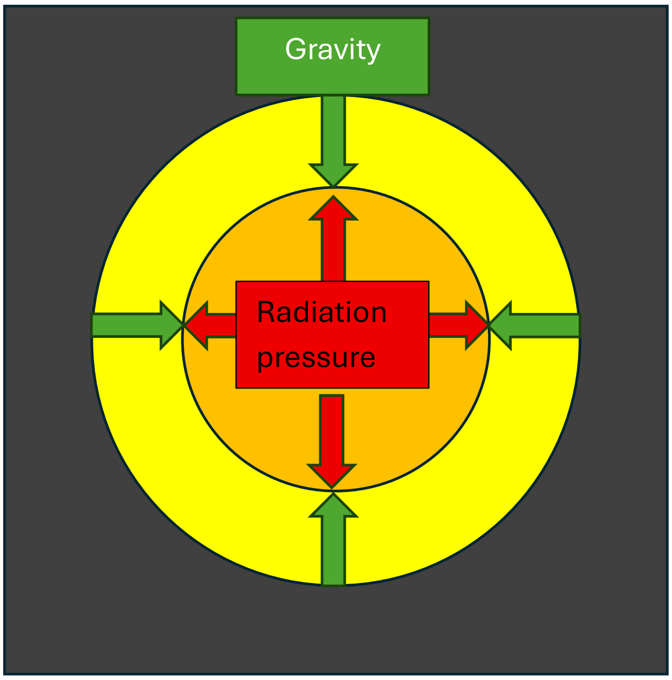


Figure 48 Opposing forces of gravity and hydrostatic pressure

14A.064 Evolution of a Star

The Sun is a typical star. It would have taken about 1000 years to coalesce (a very short time compared with the life time of stars) into a protostar of about 20 solar diameters, with a luminosity of about 100 times its present value. The evolutionary path taken by more massive stars is shorter, because there is more gravity.

The diagram (*Figure 49*) shows the evolution of stars of different masses. The letter M refers to the solar mass, so $10\,M$ is 10 solar masses. The blue lines are the time spans in years.

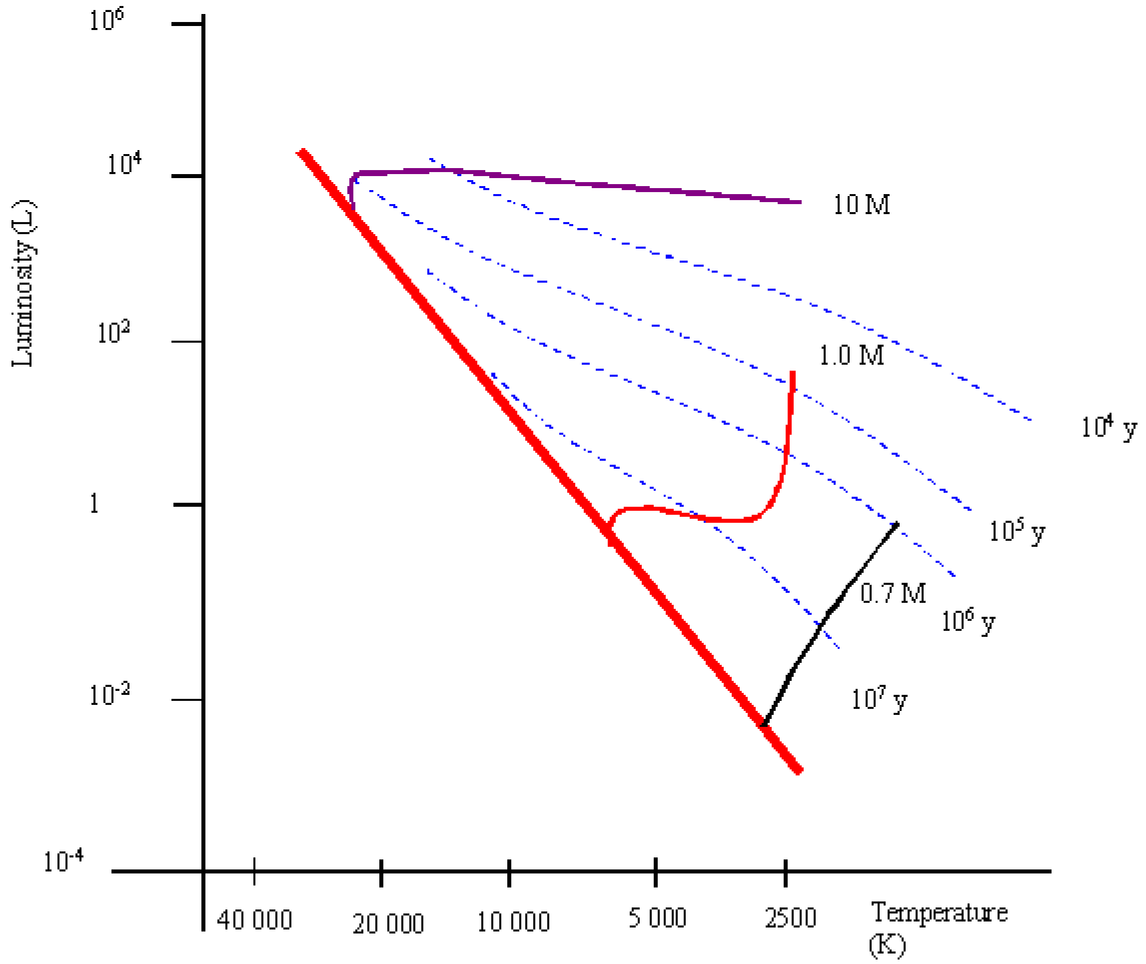


Figure 49 HR diagram showing the evolution of a sunlike star

So, let's trace how the Sun evolved before it joined the main sequence (thick red line). The luminosity would have been about 100 times what it is now for a period of about 10 000 years. Over the 8 million years the luminosity reduced to its present value. The temperature was relatively low, about 3000 K. Finally, over a period of about 10 million years the temperature gradually rose to about 6000 K.

Stars of small mass (less than about $0.8\ M$) can take 10 000 000 years to reach the main sequence. Stars of mass $0.5\ M$ may not even reach the main sequence at all.

14A.065 Lifetime of a Star

Most stars spend most of their life on the main sequence of the HR diagram. The life of a star is governed by its luminosity and mass:

- the greater the **mass**, the longer it will before the hydrogen fuel runs out.
- the greater the **luminosity**, the sooner it will use up its supply of fuel.

Stars usually start off with 73% hydrogen, 25% helium, and 2% other elements. Our Sun has used up most of its hydrogen and has more helium than hydrogen. It is using about 4 million tonnes of hydrogen every second. The Sun has been burning for about 4500 million years. There is enough hydrogen to last the Sun another 4500 million years, so it won't go out tomorrow.

Stars of 25 solar masses last only a few million years, since they are extremely bright. Stars of less than 1 solar mass burn themselves out very slowly; a star of 0.5 M can last up to 200 000 million years.

14A.066 Cepheid Variables

Most stars show a **constant luminosity** in their lifetimes. This means that:

- their power output is constant.
- their brightness remains constant.

Some stars **pulsate**, i.e. their luminosity changes periodically from a minimum to a maximum value. They are called **Cepheid Variables**. They were given this name as they were first discovered in the constellation of Cepheus. The stars have masses of between 4 and 20 times that of the Sun. They are also stars that are later on in their lives; they are using **helium** as a fusion fuel. The **apparent magnitude** is shown on the graph (*Figure 50*).

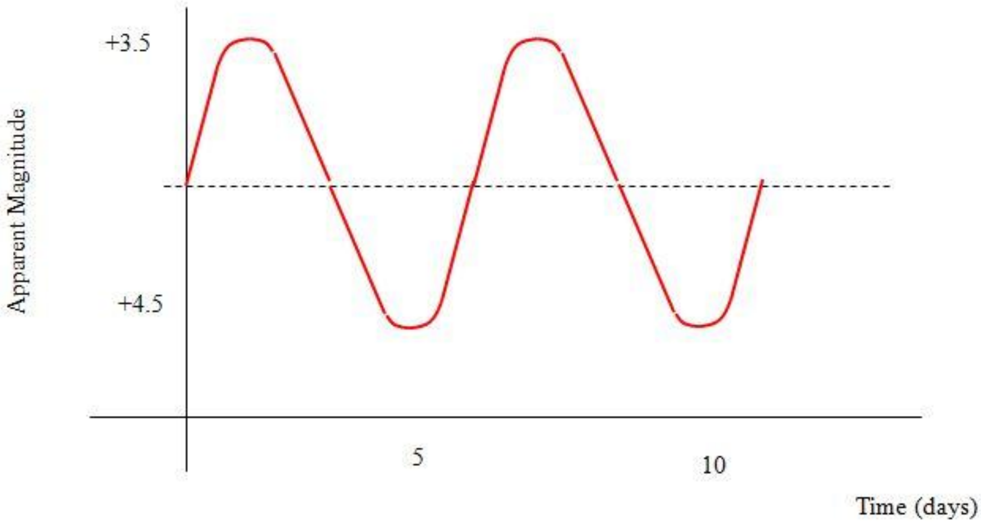


Figure 50 Periodicity of a Cepheid Variable

The Cepheid Variable shown in the graph has a period of 5 days and the magnitude varies from +4.5 to +3.5. The Sun has an absolute magnitude of +4.8. Astronomers have found that there is a relationship between the period of a Cepheid and its luminosity. There is a difference in the variation with metal rich and metal poor Cepheid Variables (*Figure 51*).

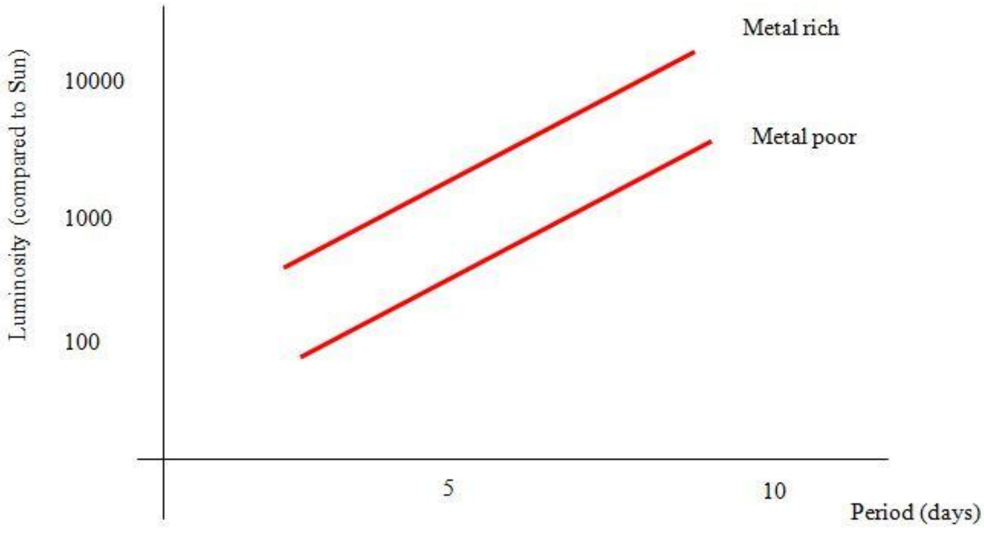


Figure 51 Variation in luminosity between metal rich and metal poor Cepheid Variables

The best theory as to why there is this variation in luminosity is that the outer envelope of the star has a lot of helium in it. We know that the helium atom is two protons, two neutrons and two electrons. At high temperatures, helium exists as helium He^{+} ions. At even higher temperatures, the second electron is removed to form He^{2+} ions, a **plasma**.

The theory is that He^{2+} is more **opaque**, which means that less light gets through. The light interacts with the helium to heat it up. As the star gets hotter, the helium expands. As it expands, it becomes cooler. The He^{2+} ions pick up electrons, to become He^{+} ions, which are more transparent. The star gets brighter. The cooler shell around the star contracts as it cools, and the process starts again (*Figure 52*).

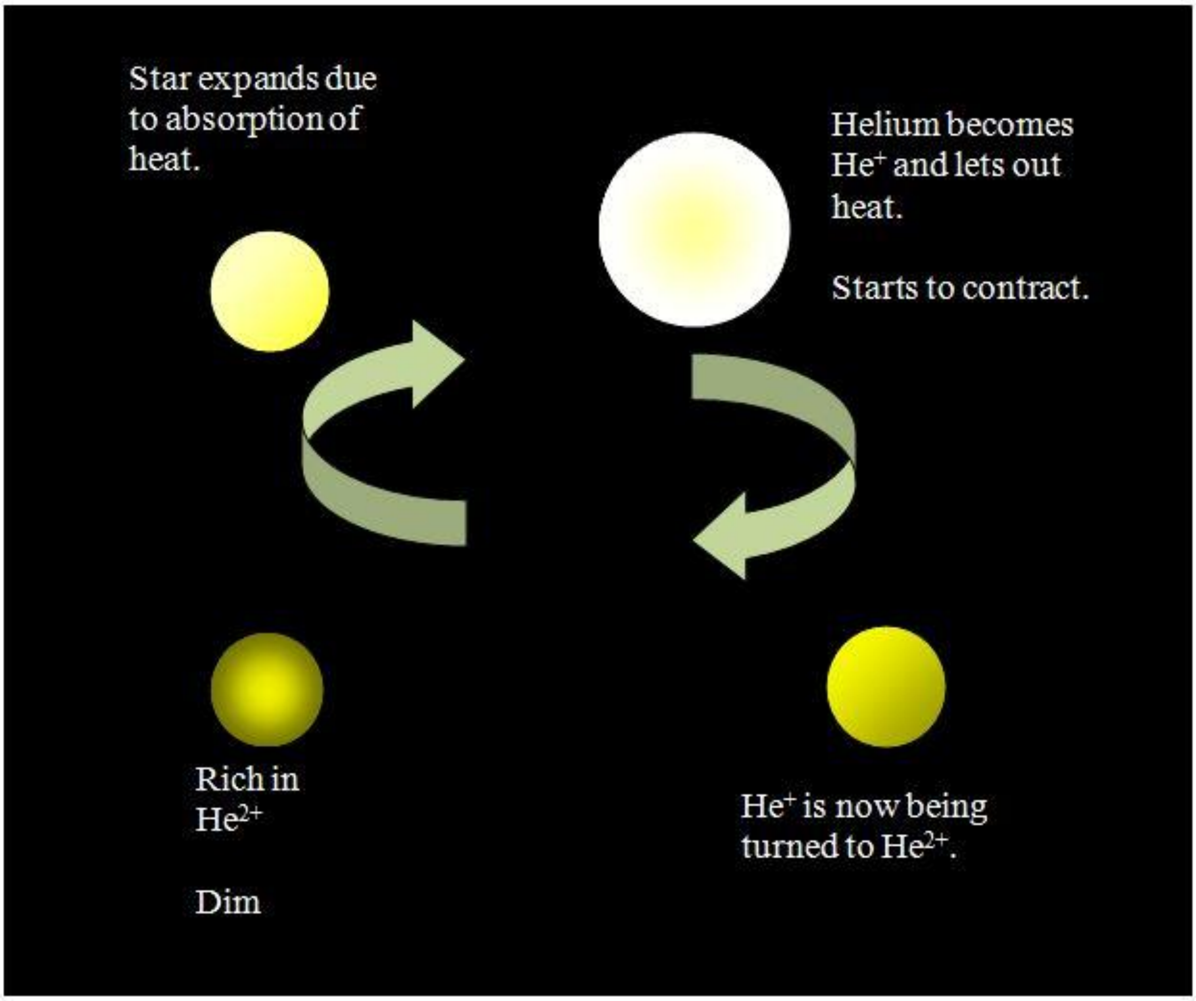


Figure 52 How it's thought that a Cepheid Variable changes luminosity

A Cepheid Variable with a period of 5 days has a **luminosity** that is about 1000 times that of the Sun.

From **Tutorial 14A.04** we saw that:

$$\Delta m = 2.5\lg\left(\frac{I_A}{I_B}\right)$$

..............*Equation 54*

We can use this equation to give a **difference in magnitude**:

$$\Delta m = 2.5 \lg 1000 = 7.5$$ .... *Equation 55*

Then we use:

$$\Delta m = 5\lg\left(\frac{d}{10}\right)$$

.... *Equation 56*

Then putting numbers in and re-arranging:

$$\log d - \log 10 = 7.5 \div 5 = 1.5$$ .... *Equation 57*

$$\log d = 1.5 + 1 = 2.5$$ .... *Equation 58*

$$d = 10^{2.5} = \underline{\mathbf{320\ pc}}$$

Cepheid variables have well-defined luminosities and are referred by astronomers as **standard candles**. Their absolute magnitudes do not vary with age or distance.

14A.067 Dying Star

The **centre of a star** is where the **fusion** takes place. The outer regions are not hot enough and there is still hydrogen in a shell about the core. There is no convective circulation of gases into the core.

As fusion dies down, the expansive pressure reduces, and gravity pulls the gases in. They heat up and the pressure on the helium in the core rises. **Helium nuclei** fuse to form heavier elements. **Hydrogen fusion** increases in shells outside the core. Therefore, there is more helium and the core expands.

Meanwhile while the **outer shell** where there is hydrogen fusion moves outwards, and the star swells. The star becomes a giant. The core and the hydrogen fusion shell are relatively small, while the majority of the space of a red giant is taken up with a low density envelope. This is shown in *Figure 53*.

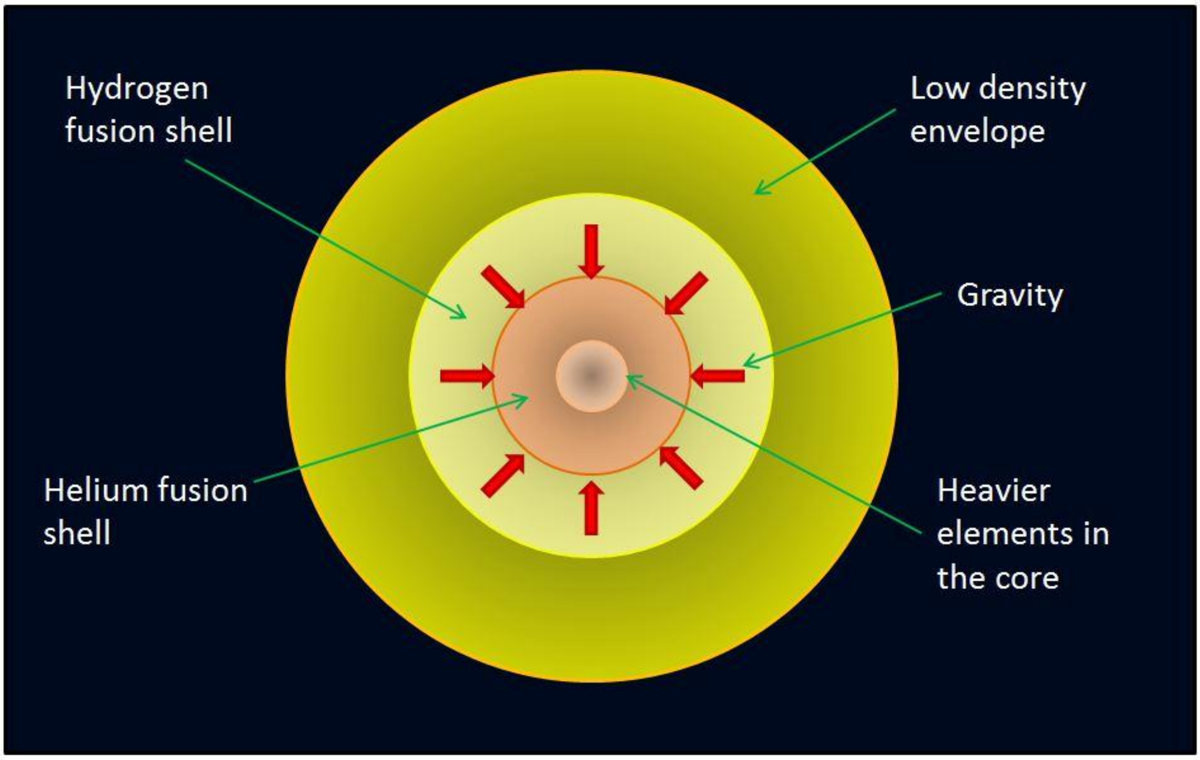


Figure 53 A dying star

When the Sun has reached this stage, all the inner planets would have been engulfed and fried to a crisp. Jupiter and Saturn will lose their gas layers to reveal their rocky cores.

In the Sun, elements like **carbon** and **oxygen** will be formed in the core. In more massive stars the conditions in the core will be sufficiently extreme further fusion will take place so that, for example, **silicon nuclei** will fuse to form **iron** (the most stable nuclide).

14A.068 Death of a Star

Eventually gravity will overcome the expansive force. In small stars, there is convection so that all the hydrogen fuses. However, the temperature never gets hot enough for helium fusion to happen. The star collapses under gravity to become a **white dwarf**. The volume is about the same as that of the Earth. Eventually it cools to a **black dwarf**, a forlorn lump doing diddly-squat in space.

The answer you got shows the enormous density, about 1 million tonnes per cubic metre. 1 cubic centimetre would have the mass of 1 tonne; if you dropped it on your foot, it would bring tears to your eyes.

For Sun like stars, death is more spectacular. The star expanding in the outer layers and contracting at the core. The radiation pressure acting outwards pushes the outer layers away from the core to form a **planetary nebula**, a ring of gas that glows brightly because of the intense radiation from the core (*Figure 54*). The material packs into an ever smaller volume until a **limit is reached** (determined by a quantum mechanical effect called **electron degeneracy**). The mutual repulsion of electrons helps to explain why your hand does not sink into a table top when you lean on it. However, under extreme pressure, electrons can be forced even closer, but there is a minimum distance to which you can force electrons together. This occurs below the **Chandrasekhar Limit**, which is about 1.4 times the mass of the Sun.

The star becomes a white dwarf, which gradually cools down.

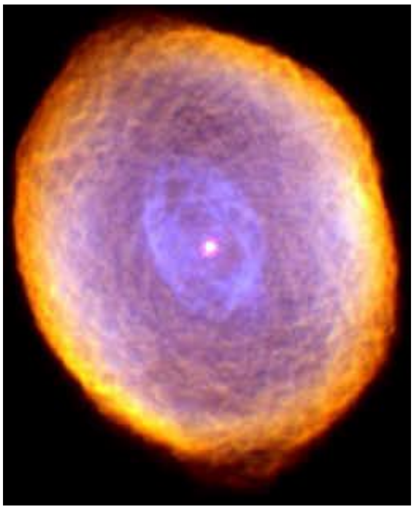


Figure 54 A planetary nebula (Photo NASA)

The picture below (*Figure* 55) shows a radio frequency image of a dying star.

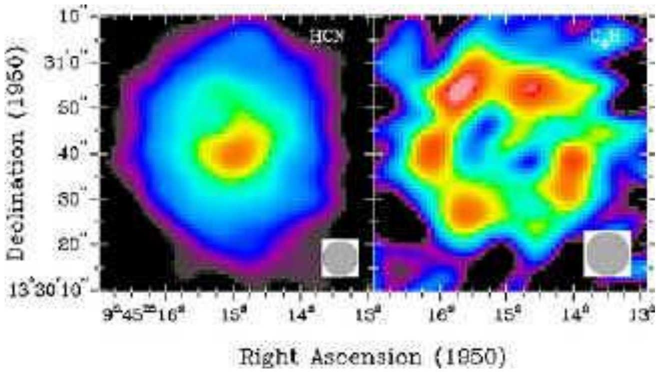


Figure 55 Radio frequency image of a dying star (Photo NASA)

14A.069 Novae and Supernovae

With more massive stars, where the core has a mass of more than 1.4 solar masses, the limit is no longer observed and electrons start to combine with protons to form neutrons, releasing neutrinos and causing the core to **collapse** in on itself. This occurs above the **Chandrasekhar Limit**, named after an Indian astrophysicist, Subrahmanyan Chandrasekhar (1910 - 1995). The collapse takes less than 1 second, and the density rises to about 4 x 10^{17} kg m^{-3}. We have a **neutron star**. Like electrons, there is a limit to which neutrons are squashed together, determined by **neutron degeneracy**. The interaction is:

$$p + e^- \rightarrow n + \nu_e$$

Just before a supernova explosion, a shower of neutrinos, X-ray, and gamma photons may be observed. This is because the neutrinos released by this process and nuclear fusion propagate much faster than the shockwave passing through the body of the star.

The outer layers may also collapse in to collide with the dense core, which can no longer be compressed. The material bounces back as a shockwave, which takes a few hours to propagate through the material. The material is torn away in a titanic explosion called a **supernova** (*Figure 56*). The absolute magnitude may be -15 or even -20.

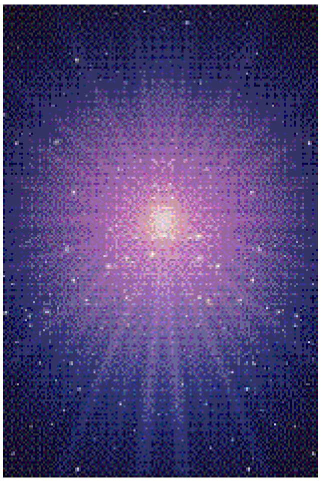


Figure 56 A supernova (Photo NASA)

Often there is a cloud of gas, a **nebula**, which propagates away from the site of the explosion. This can be seen in the picture below (*Figure 57*).

Figure 57 A nebula resulting from a supernova explosion (Photo NASA)

Iron is the largest element that can be caused by fusion. It is made in the largest of stars. It requires the extreme conditions of a supernova to make elements that have nucleon numbers greater than 56. So copper, with a nucleon number of 64, will have been made in a supernova explosion, as have many of the other rare elements in your computer.

It is thought that supernovae emit an audio-frequency hum just before they explode.

14A.0610 Classification of Supernovae

As with any star, we can analyse the light by looking at the **absorption spectrum**, and much can be learned about the explosions from the chemical compositions. Astronomers classify the supernovae like this:

- Type I supernovae have no strong hydrogen lines. They are subdivided into:
  - Type Ia - contains a strong line of silicon. Their luminosity rises rapidly to about 10^9 times that of the Sun, then decreasing smoothly and gradually. They arise when a white dwarf attracts material from a companion star. Carbon fuses to form silicon in an unstoppable fusion reaction. The white dwarf explodes.
  - Type Ib - contains strong lines for helium. They are formed by the collapse of supergiant stars with no hydrogen. The light output decreases smoothly and gradually.
  - Type Ic - contain no strong lines for hydrogen, helium, or silicon. They occur when supergiant stars with no hydrogen or helium collapse. The light output decreases smoothly and gradually.
- Type II supernovae have strong lines for hydrogen. They are supergiant stars which still have hydrogen and helium in the outer layers as they collapse. They do not have such intense luminosity. The light output fades gradually but unsteadily.

14A.0611 Type Ia Supernova

In binary star systems one of the stars becomes a white dwarf. This is shown as Star B in the picture below. The white dwarf has a mass of about 1.4 times that of the Sun and has a large amount of carbon and oxygen as fusion products. At this stage the force of gravity is not strong enough to cause collapse by electron degeneracy (where electrons cannot be packed any tighter than they are now). Material from Star A is attracted by the gravity of Star B and is pulled away from Star B. It lands on Star B and collects (**accretes**). However, as material from the Star A accretes, the temperature rises high enough for carbon fusion to happen on Star B. Carbon-carbon fusion results in magnesium.

See *Figure 58*.

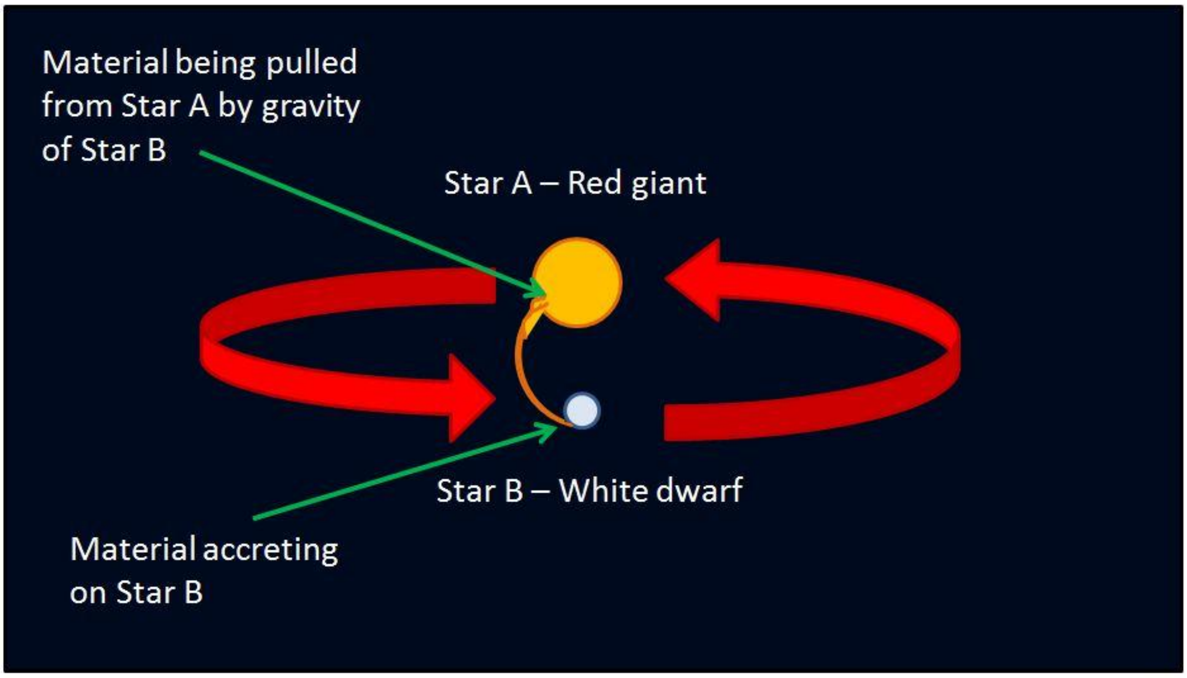


Figure 58 Material being attracted from a red giant to a white dwarf

Eventually the carbon fusion will result in a white dwarf that consists of magnesium, oxygen, and neon. The white dwarf will collapse under gravity. **Neutron degeneracy**, where electrons are forced into protons making neutrons, will cause a neutron star to be formed.

In rare cases the binary system consists of two white dwarfs like this (*Figure 59*).

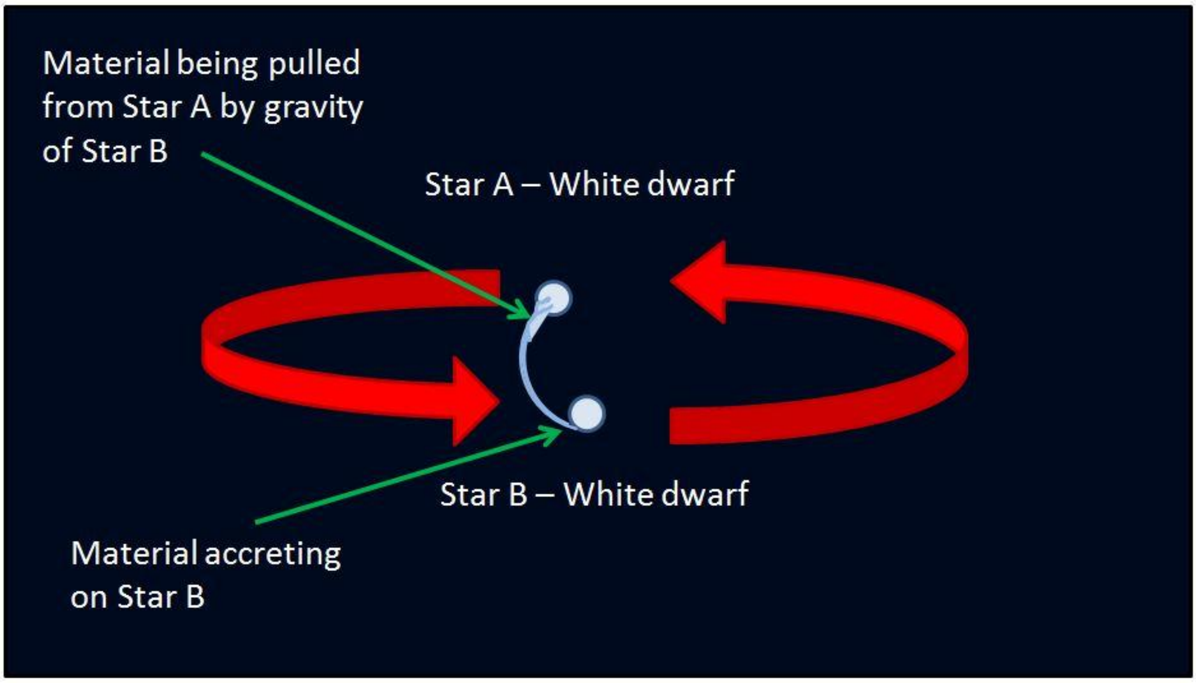


Figure 59 Material being attracted from one white dwarf to another

In this case, the amount of material that accretes does not quite reach the critical value of 1.44 solar masses. The temperature of the core rises as the pressure and density rises. Strong convection currents occur in the atmosphere, a stage that is estimated to last about 1000 years. Although it's not known how it starts, a carbon fusion flame ignites and spreads. It sets off oxygen fusion. In a main sequence star, the process would be regulated by thermal expansion, but in a white dwarf, this does not happen, and the rise in temperature runs away. The fusion events last only a few seconds, releasing vast amounts of energy (about 10^{44} J) to tear the star apart. The shock wave propagates at speeds of up to 20 000 km s^{-1}. This blows away any remaining material of the companion star. The process is different to normal supernovae in that the core does not collapse.

The **absolute magnitude** of such an event is M = 19.3, with little variation. This enables astronomers to use Type 1a supernovae explosions as standard candles, giving a good reading of the distance of the galaxies from the Sun.

Initial analysis of spectra reveal elements such as oxygen and calcium. As time progresses, heavier elements are detected, such as iron, which is the largest element that can be formed by fusion. Other elements of nucleon number 56 also arise, such as nickel-56. This isotope decays by beta decay through cobalt-56 to iron-56, releasing high energy gamma photons, and photons released by the interactions of high speed electrons with matter (*Figure 60*).

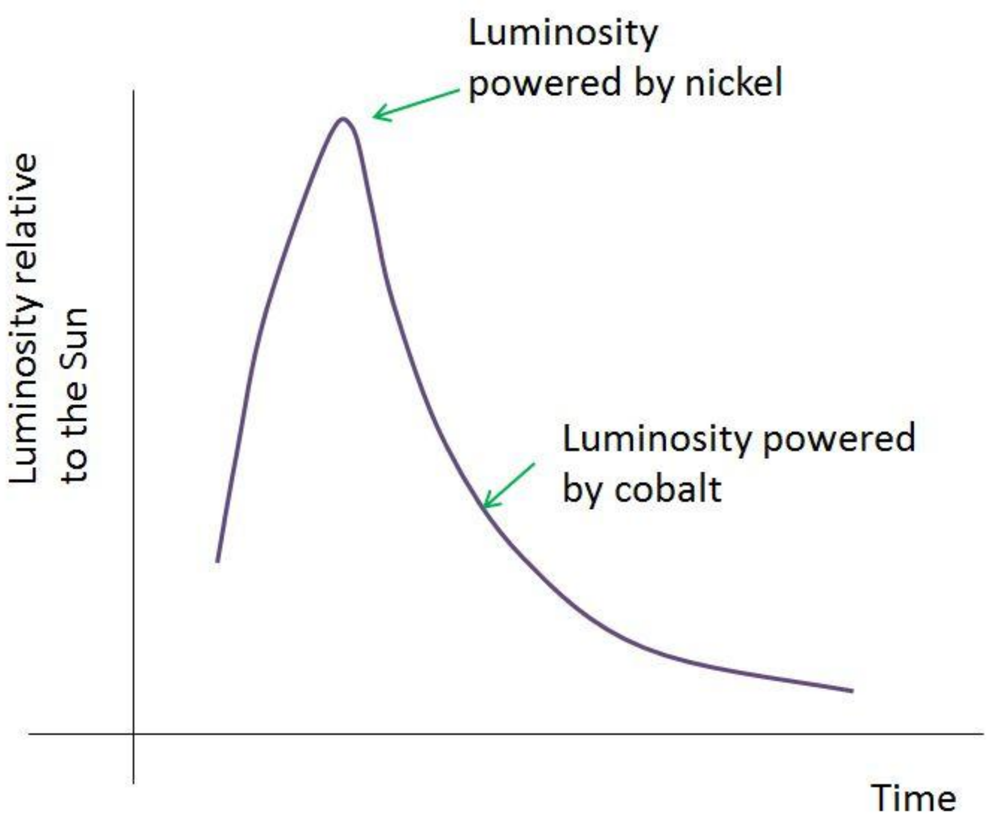


Figure 60 Peak in luminosity occurs when fusion results in the formation of nickel

All reactions in a star are fusion. Although it might be tempting to assume that carbon and oxygen react to form carbon dioxide, this does not happen. Chemical reactions involve the outer electron shells. In the extreme conditions in a star, all electrons are removed, so that the elements exist as a plasma. Once elements have cooled sufficiently to attract electrons into shells, only then can they form chemical bonds.

Type Ia Supernovae are rare events. It is thought that they represent the first kind of supernova explosions that were the end of the earliest stars. They are very distant objects.

14A.0612 Neutron Stars

The resulting neutron star is very small compared to the original star. A sun-like star would give a neutron star of diameter 30 km. The material in the Earth would be contained in a ball about 200 m across that would fit on top of your school (and squash it flat).

Some neutron stars rotate rapidly and give of beams off radiation with a regular period (*Figure 61*). They are called **pulsars**. The period can be as short as 33 ms, i.e. a frequency of 30 Hz.

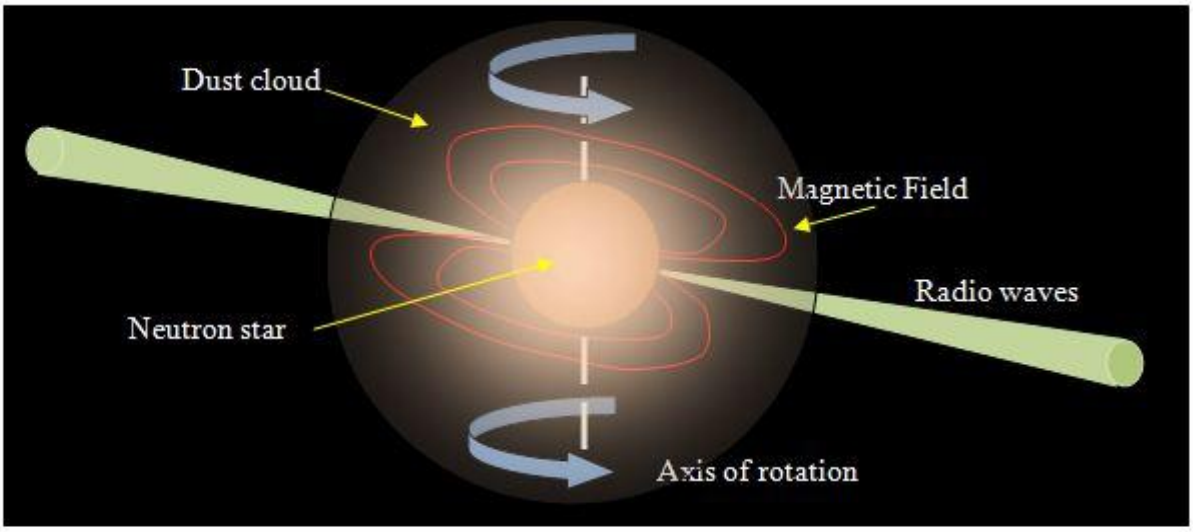


Figure 61 A pulsar

Such a star will be spinning at a rate of 900 times a minute. When a **pulsar** was detected using a radio telescope in 1967, the astronomers thought initially that the regularity of the pulse suggested a civilisation. They dubbed the pulsar LGM-1 (Little Green Men - 1). The radio waves come from the interaction of charged dust particles with the magnetic field.

14A.0613 Black Holes

In **very massive stars** the core can collapse so that even the **limit** determined by **neutron degeneracy** is broken. The gravity is so great that the core keeps on collapsing, shrinking away, according to the theory, so that it is no more than a point in space. This point is called a **singularity**, and the laws of Physics no longer apply. The gravity field in a **black hole** is so strong that even light cannot escape.

The star is surrounded by an **event horizon**, inside which nothing can be seen. It is the boundary at which light cannot escape. Abandon hope all ye who enter here.

You cannot see a black hole. But you can tell where there's a black hole as jets of high energy particles are ejected (*Figure 62*).

Figure 62 Jets ejected by a black hole (Picture from NASA, Wikimedia Commons)

A passing star is gobbled up by a black hole (*Figure 63*):

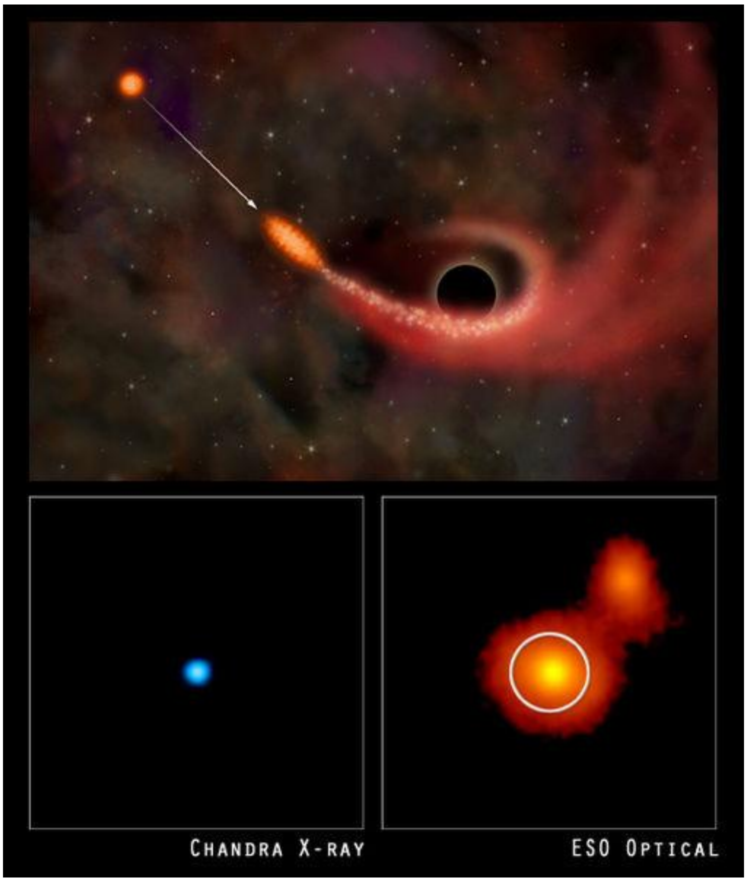


Figure 63 A star is consumed by a black hole (Picture from NASA, Wikimedia Commons)

Astronomers believe that at the centre of most galaxies is a **super-massive black hole**, a black hole of mass of 10^8 times the mass of the Sun. Black holes have such high gravity that light can be bent. This is called **gravitational lensing** (*Figure 64*).

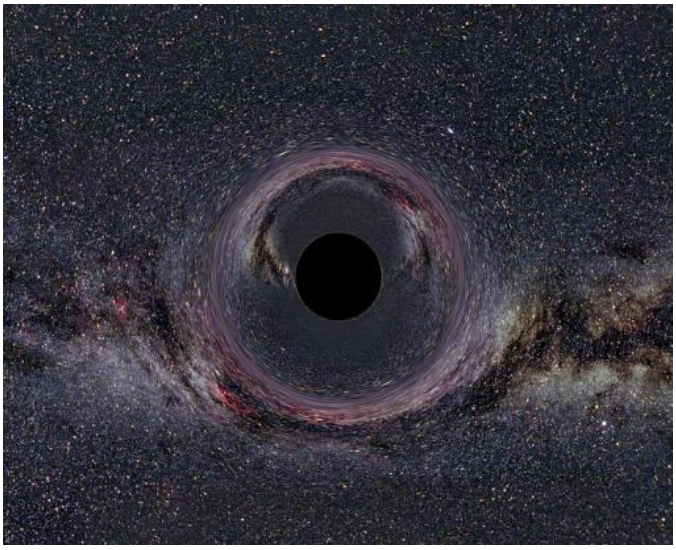


Figure 64 Gravitational lensing (Picture by Gallery of Space Time Travel, Wikimedia Commons)

14A.0614 Schwarzschild Black Hole

The Schwarzschild black hole is the simplest black hole:

- It does not rotate. Therefore, the angular momentum is zero.
- It has no electric charge.
- It exists in spacetime with no other mass.

The diagram shows the structure (*Figure 65*).

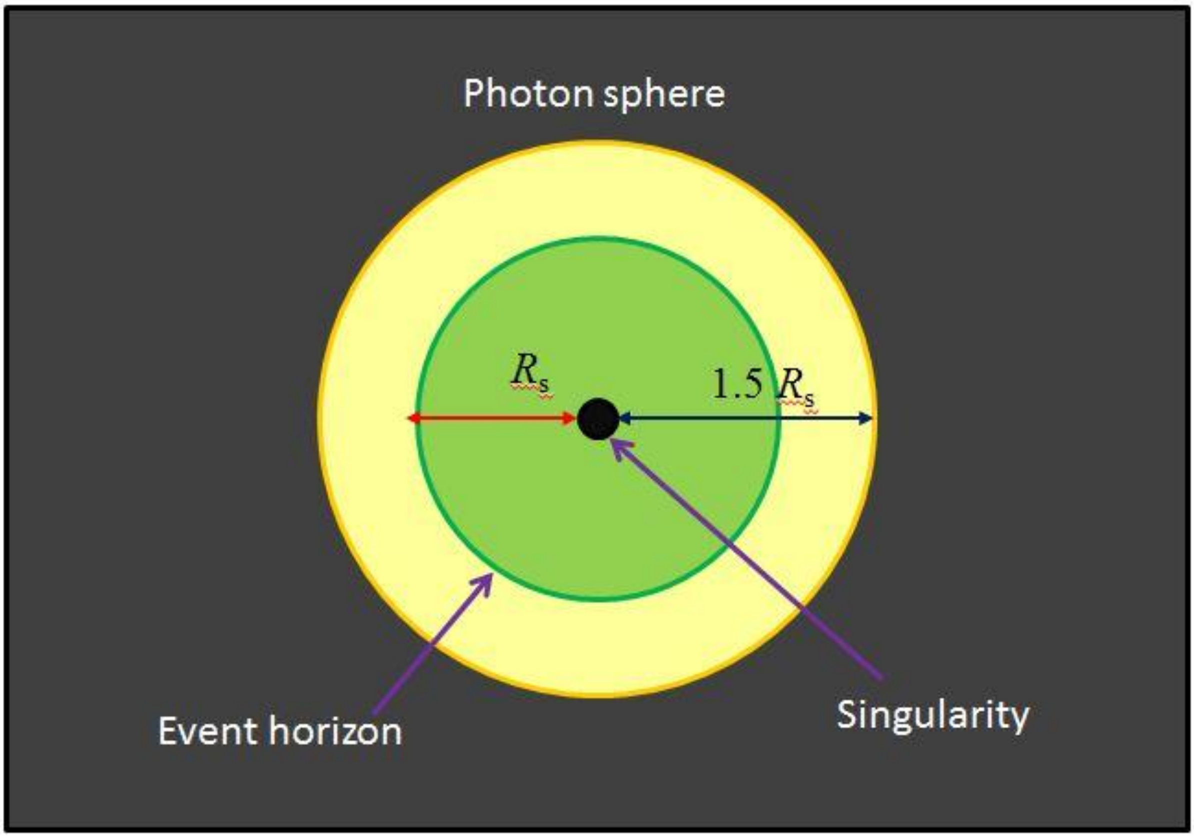


Figure 65 The simplest black hole

The features are:

- A **singularity** where space and time have infinite curvature. This is not easy to imagine. Another way is to say that the normal laws of physics do not apply.
- The **event horizon**, which is boundary of the black hole. The event horizon occurs at the Schwarzschild radius, R_s.
- The **photon sphere**. This is a region where **gravity** is so strong that light travels in circles. It is at about 1.5 times the Schwarzschild radius. A photon in this region can leave the back of your head, and it would loop round so that you could see the back

of your head. (I would see my bald patch, and an unsightly lump on the side of my neck.)

The radius of the **event horizon** is called the **Schwarzschild Radius**, code R_S. From Topic 9 (Fields), the escape velocity of a rocket can be worked out by:

kinetic energy = gravitational potential energy

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

.............*Equation 59*

Rearranging gives:

$$v^2 = \frac{2GM}{r}$$

.... *Equation 60*

The escape velocity of light can be similarly worked out:

$$c^2 = \frac{2GM}{R_S}$$

.... *Equation 61*

So, the Schwarzschild radius is given by:

$$R_S = \frac{2GM}{c^2}$$

.... *Equation 62*

Terms:

- R_S - Schwarzschild radius (m);
- G - universal gravity constant 6.67 x 10^{-11} N m^2 kg^{-2}.
- M - mass of the star (kg).
- c - speed of light (m s^{-1}).

Worked Example A star of mass 2 x 10^{31} kg forms a black hole. What is the Schwarzschild radius? What is its density in the region bounded by the event horizon?
Answer Use: $$R_S = \frac{2GM}{c^2}$$ R_S = (2 x 6.67 x 10^{-11} N m^2 kg^{-2} x 2 x 10^{31}kg) ÷ (3 x 10^8 m/s)^2 R_S = **29 600 m** (29.6 km) Density = mass/volume Volume = 4/3 x π x (29600 m)^3 = 1.09 x 10^{14} m^3 Density = 2 x 10^{31} kg ÷ 1.09 x 10^{14} m^3 = **1.8 x 10^{17} kg m^{-3}**.

There are other types of Black Hole:

- Kerr black hole.
- Reissner-Nordstrom black hole.

We will not consider these here.

The picture below summarises the life of a sun-like star (*Figure* 66).

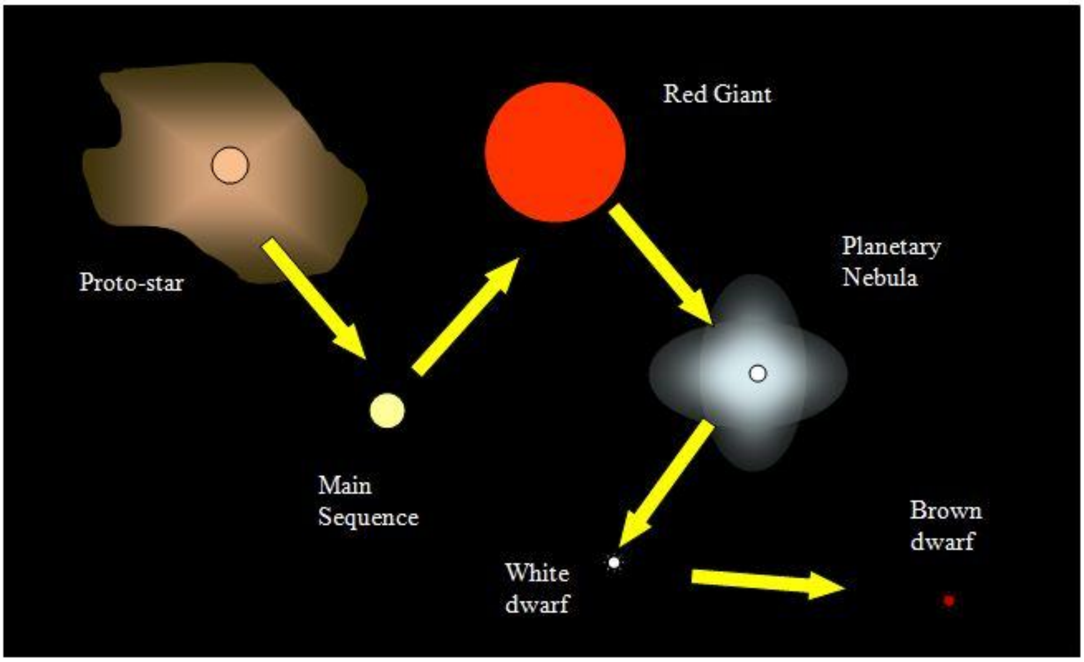


Figure 66 Summary of the life of a sun-like star

And the picture here (*Figure 67*) summarises the life of a star that is much bigger than the Sun:

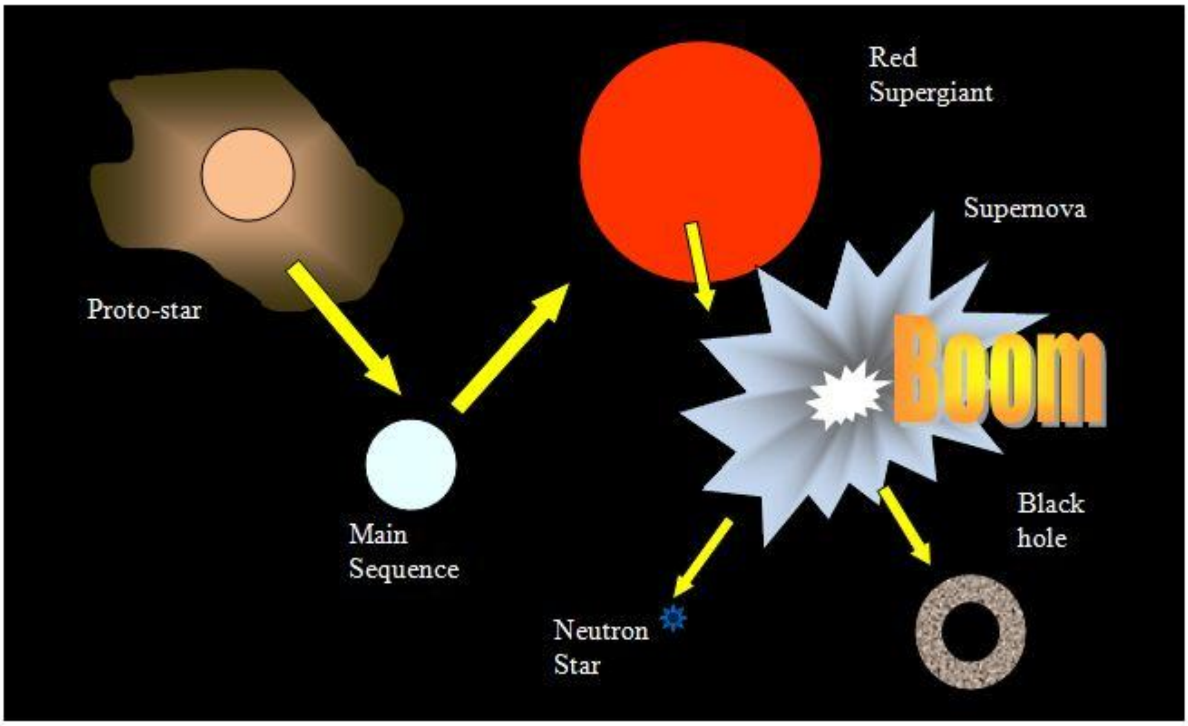


Figure 67 Life of a star much larger than the Sun

The fate of the star following a supernova explosion depends on its size. The largest stars end up as black holes.

Questions

Tutorial 14A.06

14A.06.1

Complete the table. One has been done as an example.

Star	***Luminosity (Sun = 1)***	***Surface Temp (K)***	***Group***
Sun	1.0	5800	Main sequence
Betelgeuse	20 000		Red Supergiant
Aldebaran	200	4700	
Regulus		14000	Main Sequence
Rigel	20000		Main Sequence
Sirius B	0.002	20000	

14A.06.2

Refer to *Figure 49*. How would you describe the formation of a star that was much bigger than the Sun?

14A.06.3

How many light years is 320 pc?

14A.06.4

A white dwarf has a mass of 0.2 solar masses. Use the data below to calculate the density of a white dwarf, assuming it's the same size as the Earth. Compare it with the density of the Earth and the Sun.

Data:

Mass of sun = 2.0×10^{30} kg.

Mass of earth = 6.0×10^{24} kg.

Radius of Sun = 7.0×10^{8} m.

Radius of Earth = 6.4×10^{6} m.

14A.06.5

A ball of neutron star material has a mass of 6×10^{24} kg and a diameter of 400 m. Calculate:

(a) the gravitational field strength on the surface.

(b) the escape velocity of a rocket trying to leave.

14A.06.6

Much evidence is being produced suggesting that galaxies have black holes at their centres. For example, the spiral galaxy M51 may contain a black hole with a mass one million times greater than the Sun.

(i) Explain what is meant by the term event horizon.

(ii) Calculate the radius of the event horizon for the black hole in M51.

(AQA Past Question)

3. Cosmology

Tutorial 14A.07 Cosmology

AQA Syllabus

Contents

14A.071 The Doppler Effect

You will know the **Doppler effect** as the falling note of a car or train horn as it approaches, passes, and then goes away from you. See *Figure 68*.

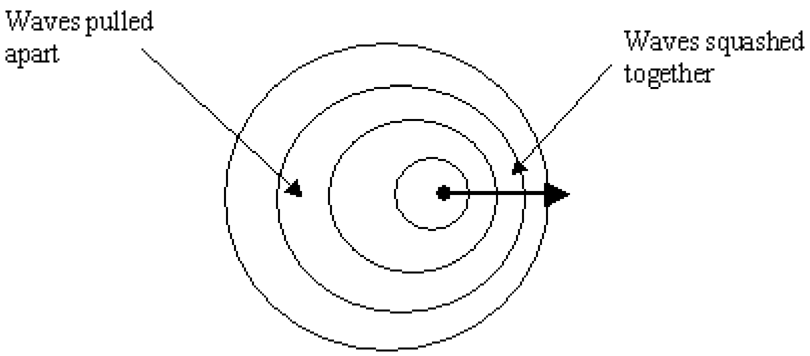


Figure 68 The Doppler effect

The importance of the Doppler effect is that it is seen with light waves and radio waves. For any object that is moving with a speed much less than that of light, it can be shown that the change in frequency is given by:

$$\frac{\Delta f}{f} = \frac{v}{c}$$

.... *Equation 63*

[Δf - change in frequency (Hz); f - original frequency (Hz); v - speed of object (m s^{-1}); c - speed of light (m s^{-1})]

Sometimes we use the term **Doppler Shift**, code z, which is the **fractional change** in frequency.

$$z = \frac{\Delta f}{f}$$

.... *Equation 64*

For wavelength the equation is similar:

$$\frac{\Delta \lambda}{\lambda} = -\frac{v}{c}$$

.... *Equation 65*

In a similar way we use the **Doppler Shift**, code z, which is the **fractional change** in wavelength.

$$z = \frac{\Delta \lambda}{\lambda}$$

.... *Equation 66*

For these equations:

- Objects moving towards the observer have a positive speed; moving away from the observer the speed is negative.
- If the object is moving away, the frequency is lower so that Δf is negative. The wavelength will be longer. Astronomers call this **red shift**.
- If the object is coming towards the observer, the frequency is higher, so Δf will be positive. The wavelength will be shorter. Astronomers call this **blue shift**.

The Doppler shift can be summed up in this table:

Doppler Shift, z	Moving towards	Moving Away
Frequency	Positive (higher)	Negative (Lower)
Wavelength	Negative (shorter)	Positive (Longer)

While we will focus on the Doppler Effect in Astrophysics, it can be used to measure **blood flow** with **ultrasound** (See Topic 14B – Medical Physics) or detect the **movement of vehicles** using **radar**. A radar gun used by the police emits **microwaves** and measures the **Doppler shift** as a car approaches. The traffic cop gets a direct readout of the speed of the car. Fixed speed cameras work the same way. If you are above the tolerance on the speed limit (about 10 %), the camera will take a picture, and you will get a speeding ticket (3 points on your licence).

Worked example The wavelength of a pale blue line in the hydrogen spectrum is 486.27 nm as measured in the lab. When the same spectral line is looked at in a star, the wavelength is now 486.94 nm. What is the speed of recession?
Answer Find out the change in wavelength: $\Delta\lambda = 486.94 - 486.27 = 0.67 \text{ nm} = 0.67 \times 10^{-9} \text{ m}$. Use: $\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$ Substitute into the equation: $0.67 \times 10^{-9} \text{ m} \div 486.27 \times 10^{-9} \text{ m} = -v \div 3 \times 10^{8} \text{ m s}^{-1}$ $v =$ **-4.1×10^{5} m s^{-1}** (Negative means away from us. Strictly speaking, it should be velocity.)

The minus sign tells us that the star is receding from us. The longer wavelength is called red shift, i.e. it has been shifted towards the red end of the spectrum. This is shown below. The top spectrum (*Figure 69*) is what we would expect for an element in the lab. The bottom spectrum shows the same pattern, but shifted towards the red.

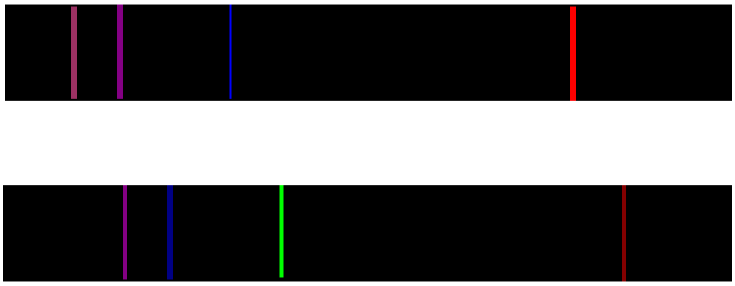


Figure 69 Red shift in the spectrum of a star receding from us

We can see that the pattern is the same, but the colours are different. The same principle as in Question 14A.7.2 is used to determine the rotation of a star.

14A.072 The Doppler Effect in Astrophysics

The Doppler effect is used in other ways:

- looking at the rotational period of stars
- rotational periods of planets.
- Orbital period of binary stars.

60 % of the stars are actually pairs, making our Sun as a single star in the minority. (It is thought that Jupiter is a failed star.) Binary stars consist of two stars orbiting about their common centre of mass. If the stars are of equal mass the orbits are like this (*Figure 70*).

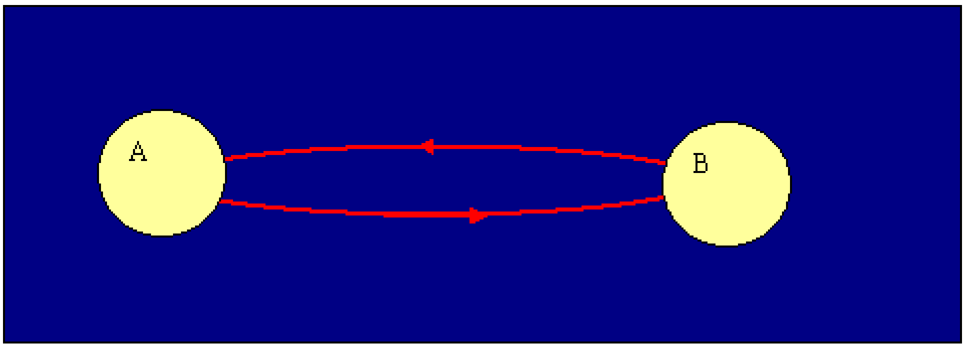


Figure 70 Two stars of equal mass orbiting each other

We can use the Doppler shift to tell us how the stars are orbiting. Let's look at a single line which we know is yellow in the lab (*Figure 71*).

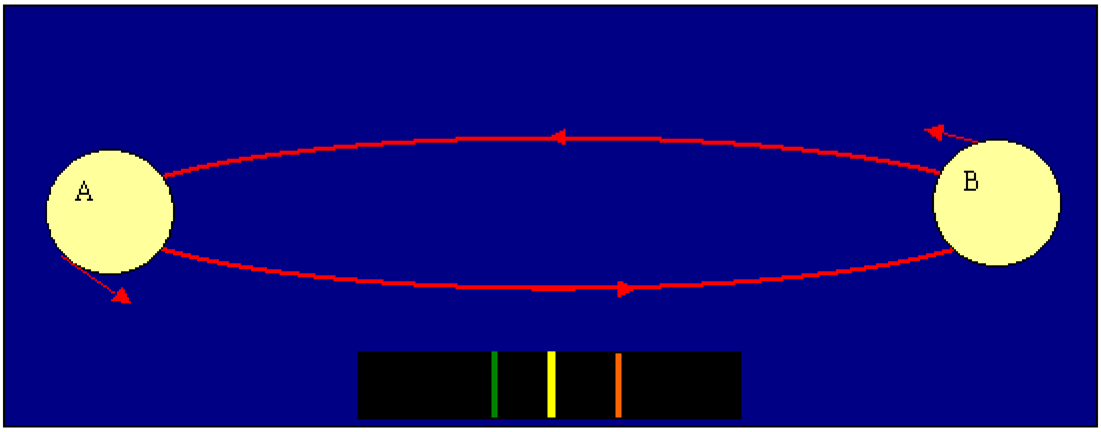


Figure 71 Doppler shift for a yellow spectral line

That line for Star A is blue shifted, which means it has a shorter wavelength, so is approaching us. That for B is red shifted which means that Star B is going away.

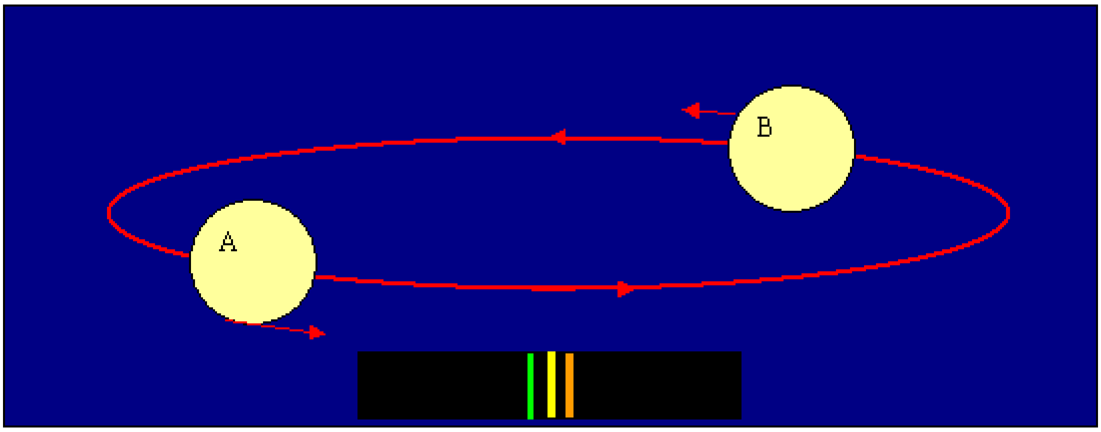


Figure 72 A few months later, the Doppler shift is less

Notice that now the stars have moved around in their orbit, the blue shift and red shift are less (*Figure 72*).

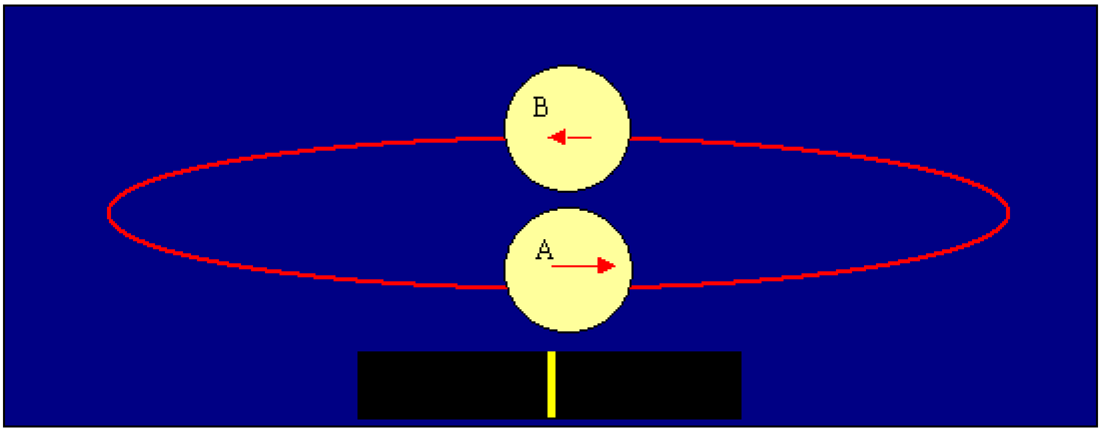


Figure 73 Stars are neither moving closer nor moving further away

When the stars are in this position (*Figure 73*), we only get the one spectral line as both stars are neither moving towards us nor away from us.

If binary stars are a long way away, it may not be possible to resolve them. However, we can still see a change in their spectroscopy. These are called **spectroscopic binaries**. These two stars are the same mass and are in the same orbit (*Figure 74*).

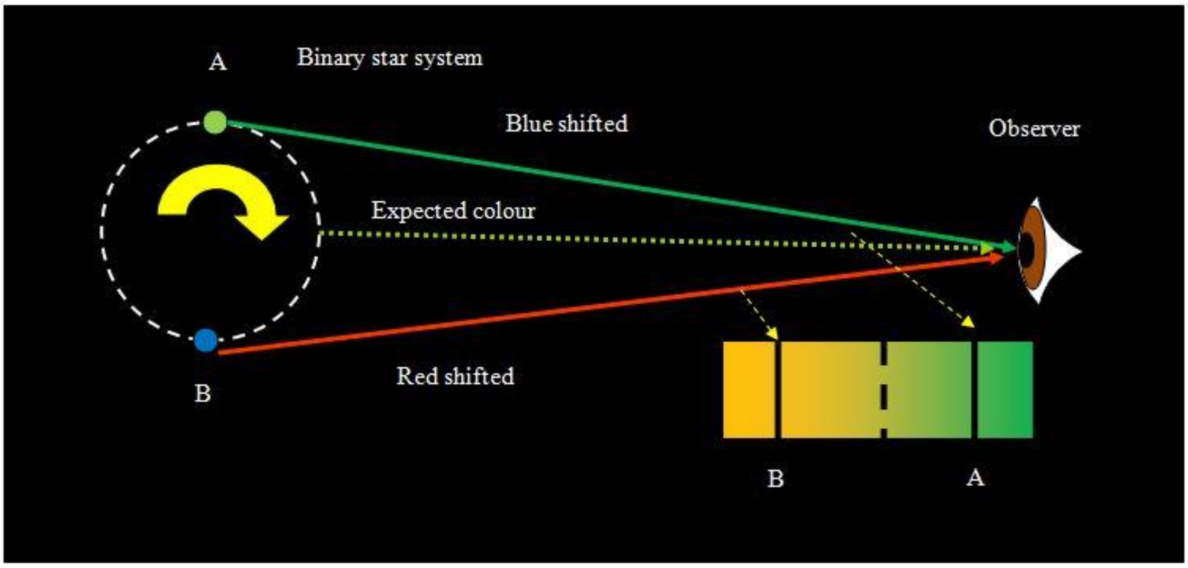


Figure 74 Spectroscopic binaries

Star A is coming towards the observer, so its light is blue shifted. Star B is moving away, so its light is red shifted. When the two stars are in line with the observer, the line on the absorption spectrum is as expected. Over a period of time, the two absorption lines would move between the middle position and the extremities.

Half an orbit later, we would see *Figure 75*.

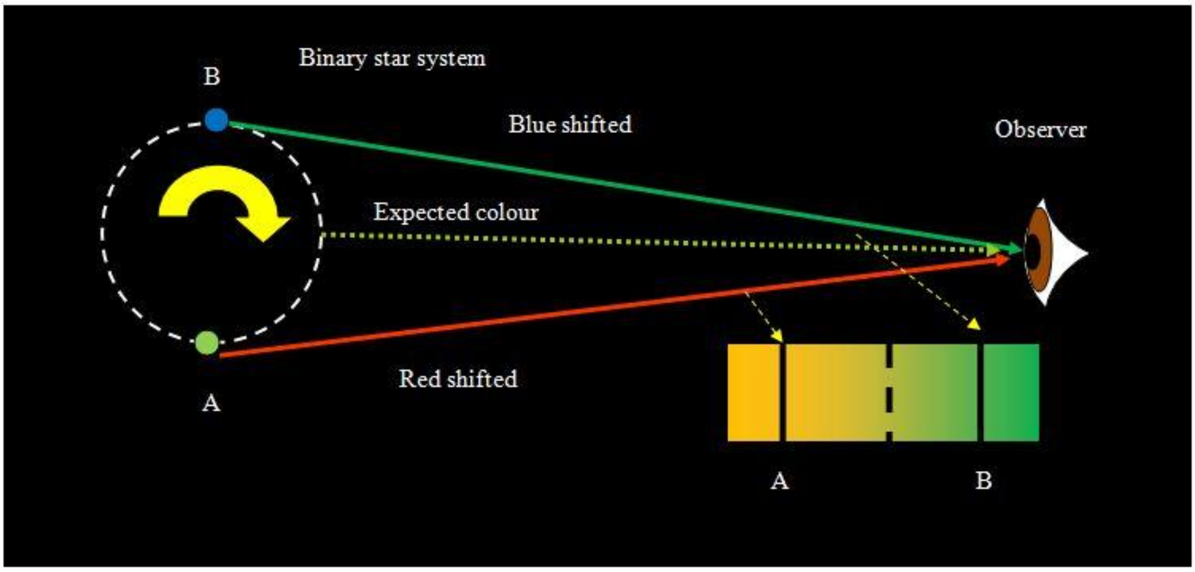


Figure 75 Half an orbit later

If the stars are of different masses, we see orbits like this (*Figure 76*).

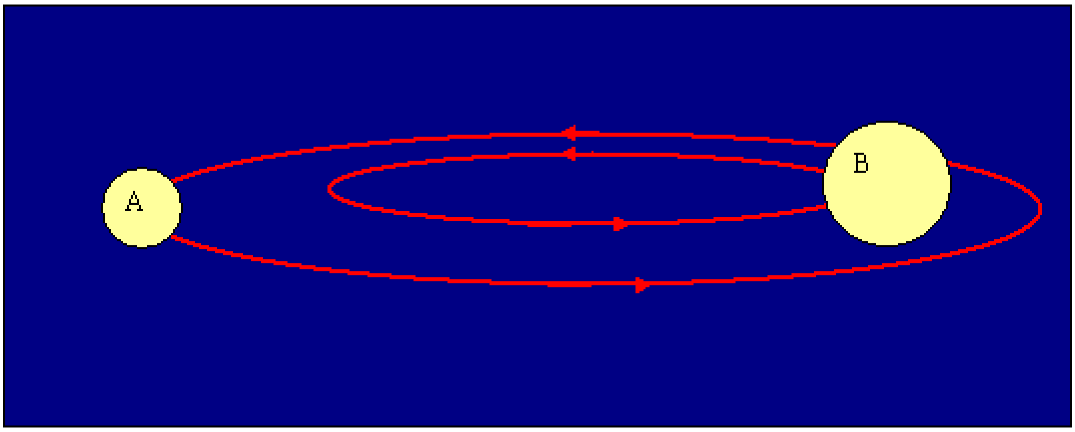


Figure 76 Stars of different mass orbiting each other

Both stars will have the same orbital period. The smaller star has a larger orbital radius, hence larger linear speed. The change in wavelength will be greater for the smaller star.

Worked Example
Two stars are orbiting each other as a spectroscopic binary. One star has more mass than the other. A spectral line of an element merges once every 1.5 years. It has a laboratory wavelength of 486 nm. The line then splits so that there is a difference of 0.042 nm and 0.024 nm. Calculate:
(a) the orbital speed of each star.
(b) the radius of each orbit.

Answer
(a) Work out the Doppler Shift, z for each star:

$$z = 0.042 \text{ nm} \div 486 \text{ nm} = 8.64 \times 10^{-5}$$

$$z = 0.024 \text{ nm} \div 486 \text{ nm} = 4.94 \times 10^{-5}$$

Work out the speed for each star.
For the smaller star:

$$v = zc = 8.64 \times 10^{-5} \times 3.0 \times 10^{8} \text{ m s}^{-1} = \mathbf{2.59 \times 10^{4} \text{ m s}^{-1}}$$

Similarly for the larger:

$$v = \mathbf{1.48 \times 10^{4} \text{ m s}^{-1}}$$

(b) Use:

$$r = \frac{vT}{2\pi}$$

$$T = 1.5 \text{ y} \times 365.25 \text{ dy}^{-1} \times 86400 \text{ s d}^{-1} = 4.73 \times 10^{7} \text{ s}$$

For the faster star:

$$r = \frac{2.59 \times 10^{4} \text{ m s}^{-1} \times 4.73 \times 10^{7} \text{ s}}{2\pi} = 1.95 \times 10^{11} \text{ m} = \mathbf{2.0 \times 10^{11} \text{ m}}$$

For the slow star:

$$r = \frac{1.48 \times 10^{4} \text{ m s}^{-1} \times 4.73 \times 10^{7} \text{s}}{2\pi} = \mathbf{7.0 \times 10^{11} \text{ m}}$$

14A.073 Exoplanets

For centuries we have wondered if there are other planets out there. A lot of science fiction is based on such planets that can support carbon-based life. It is only recently that we have seen evidence for the existence of planets outside the Solar System. Evidence was observed as early as 1917 but was not interpreted as such. The first planet that was conclusively proved to exist outside the Solar System was discovered in 1988. Now there are reckoned to be in the region of 3700 (if not more) **extrasolar planets** or **exoplanets**. Exoplanets orbit around a star (or a binary system), in the same way as the planets of the Solar System orbit the Sun.

Direct detection of exoplanets is very difficult. All planets reflect the light of their parent star, but the light is usually too faint to be observed from even the best telescopes. Also, the faint light is often swamped by the light of the parent star. Direct observation is possible if the planet is very big (bigger than Jupiter) as such planets can give out large amounts of radiation and can be detected (*Figure 77*).

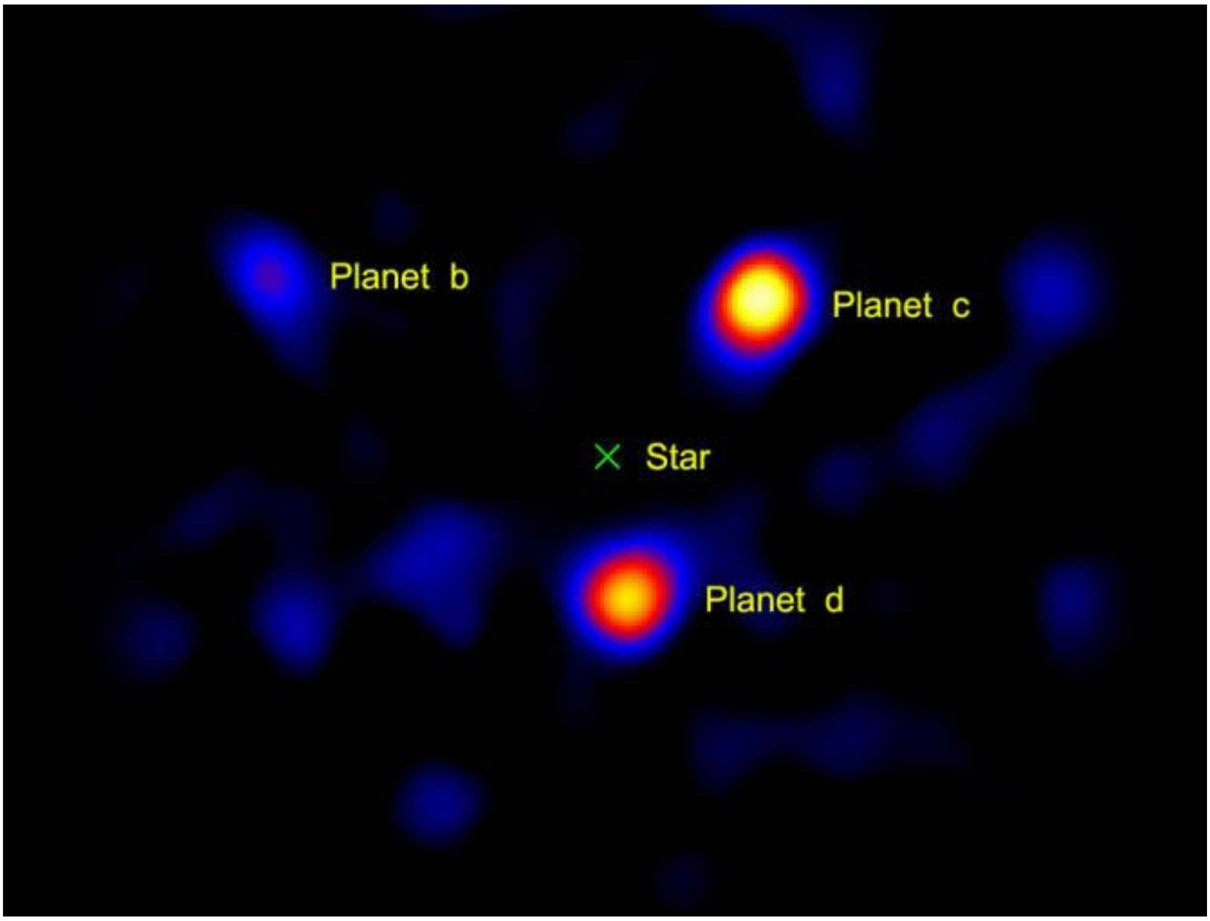


Figure 77 Direct observation of two (very large) exoplanets

There are a number of ways of indirect detection of exoplanets. The most common way is to study the **radial velocity** of a planet-star system. We tend to think of planets orbiting a fixed point. However, a planet will exert a gravitational force on its parent star, so that both will orbit around a single point of the system's centre of mass. The idea is shown in the picture below (*Figure 78*).

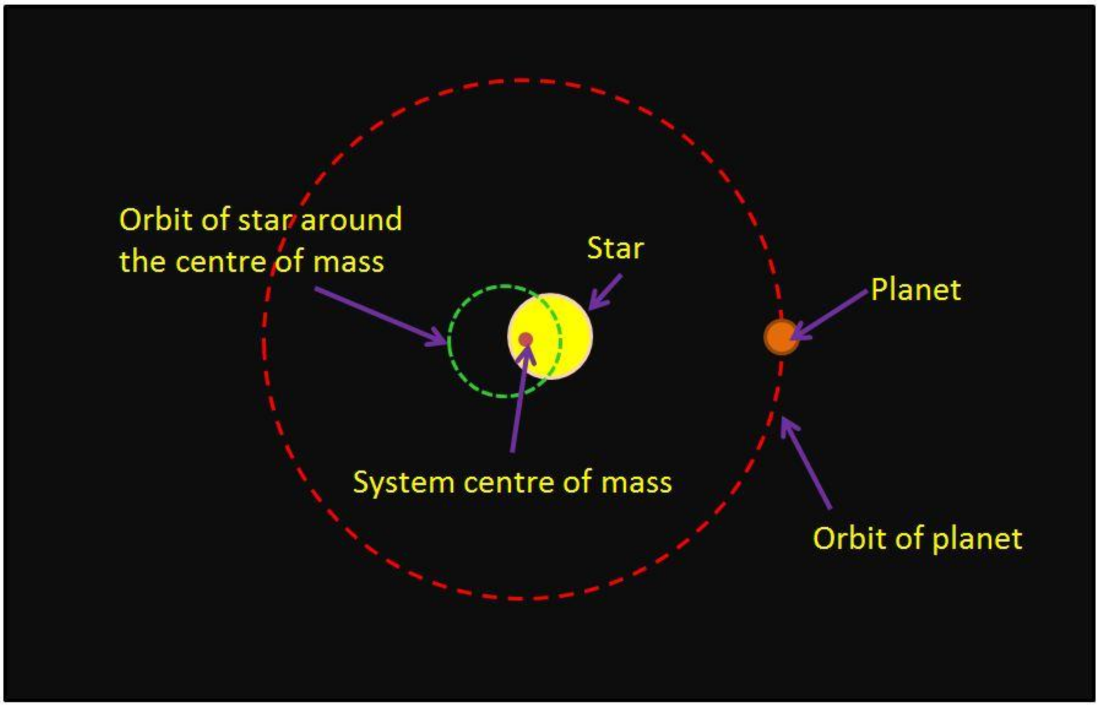


Figure 78 Effect of a planet's gravity on its host star

We don't see the planet, but instead we see the star moving from side to side. It happens to the Sun under the influence of Jupiter. The movement can be detected by Doppler Shift (*Figure 79*).

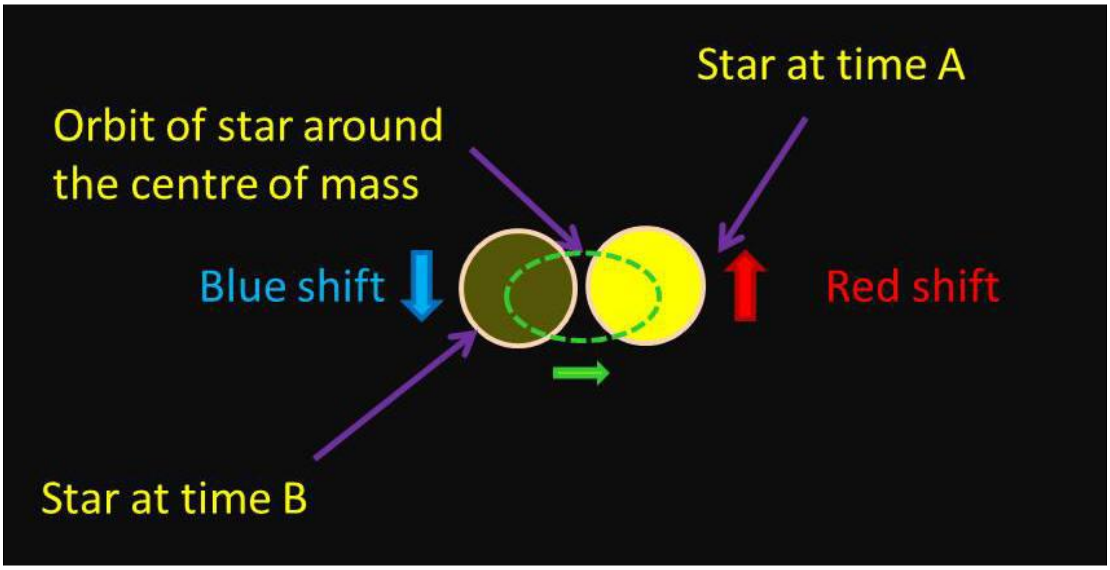


Figure 79 Doppler shift of the orbit of a star round the centre of mass of a star planet system

When the star is approaching us, spectral analysis reveals that the star is blue shifted. When it goes away from us there is red shift. The difference in speed is about 13 m s^{-1} for Jupiter and the Sun. (The Earth exerts a force on the Sun such that the difference in speed is 9 cm s^{-1}). Equipment has been made that can detect a Doppler Shift that is equivalent to a speed of as little as 1 m s^{-1}.

The variation of the speed with time is shown on a graph like this (*Figure 80*).

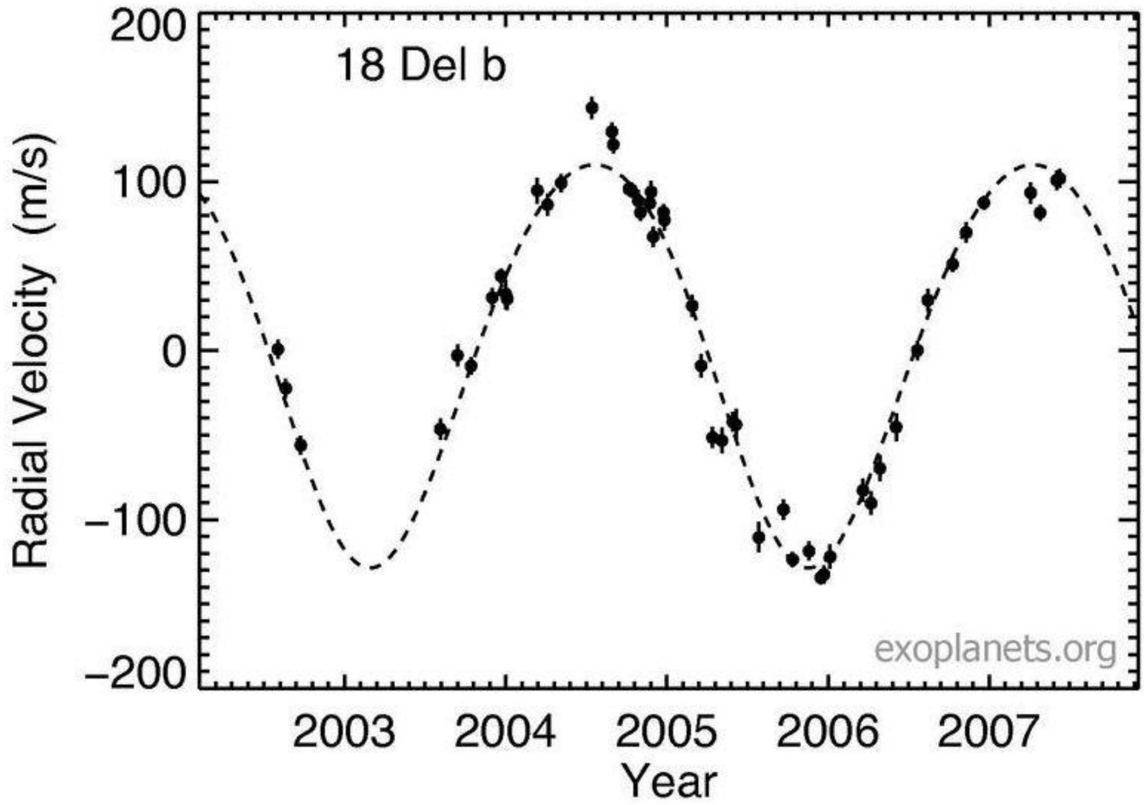


Figure 80 A plot of variation of radial velocity against time

This plot was retrieved from the Exoplanet Orbit Database and the Exoplanet Data Explorer at exoplanets.org, maintained by Dr. Jason Wright, Dr. Geoff Marcy, and the California Planet Survey consortium. In this case, the speed of approach is about 100 m s^{-1}, while the recession speed is about 140 m s^{-1}. This could show that the star is moving away from us about 20 m s^{-1}. The orbital period of the planet is about 3 years.

This method has been used to detect most exoplanets observed so far. It is a method that is suitable for planets that are relatively close to Earth, about 160 light years. Also, the stars need to be relatively low mass. Jupiter sized planets have been detected to a distance of about 10 Astronomical Units from their parent stars.

Another way of detecting the presence of an exoplanet is to observe its transit of its parent star, provided of course that its transit is in a suitable plane for us to observe it. The intensity of the light received from the star is reduced as the planet crosses its face. The diagram shows the idea (*Figure 81*).

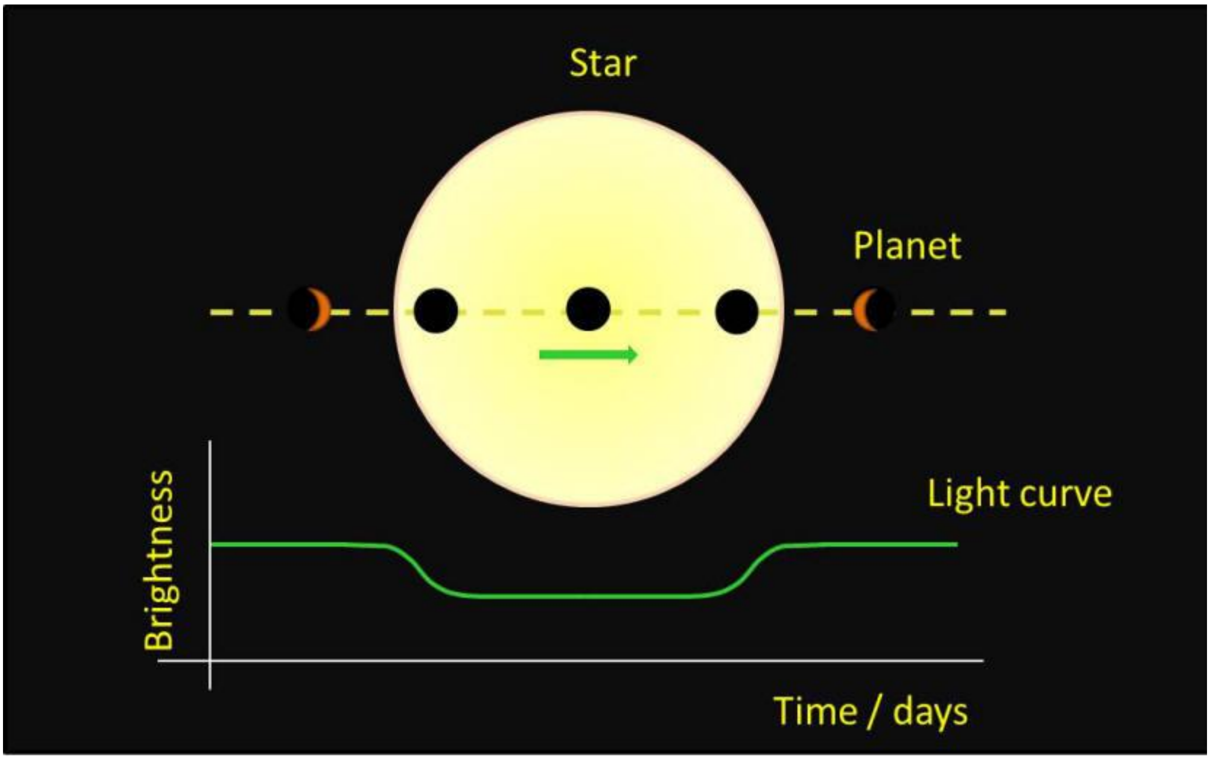


Figure 81 An exoplanet's transition across its host star

There are other methods that have detected exoplanets, but these are beyond the scope of this discussion. Here are two links you may find interesting:

https://exoplanets.nasa.gov/

https://www.space.com/17738-exoplanets.html

The hunt is on for exoplanets that have the possibility of life, i.e. have atmospheres that have a temperature of about 270 - 300 K, and show signs of water. Several of these have been detected. There is every possibility that there are life forms on other planets.

It is worth remembering that the laws of physics as we know them here on Earth would apply to **all** planets, wherever they are in the Universe. All the knowledge gleaned from our observations of the Universe are underpinned by the **Newtonian** Laws of Physics (or classical Physics). Newton's Laws were written three hundred and fifty years ago, but we use them today to send space probes to far-off places.

14A.074 Hubble's Law

An American astronomer Edwin Powell Hubble established a relationship between the **velocity** of recession of a galaxy and its **distance** from the Earth. He chose a galaxy that could be resolved into individual stars and analysed spectra against known spectra. He studied:

- the red shift.
- the speed of recession (moving away).
- the distance from the Earth by measurements of Cepheid Variables.

He found that the further the galaxy was from the Earth, the faster it was moving away.

The graph shows the idea (*Figure 82*).

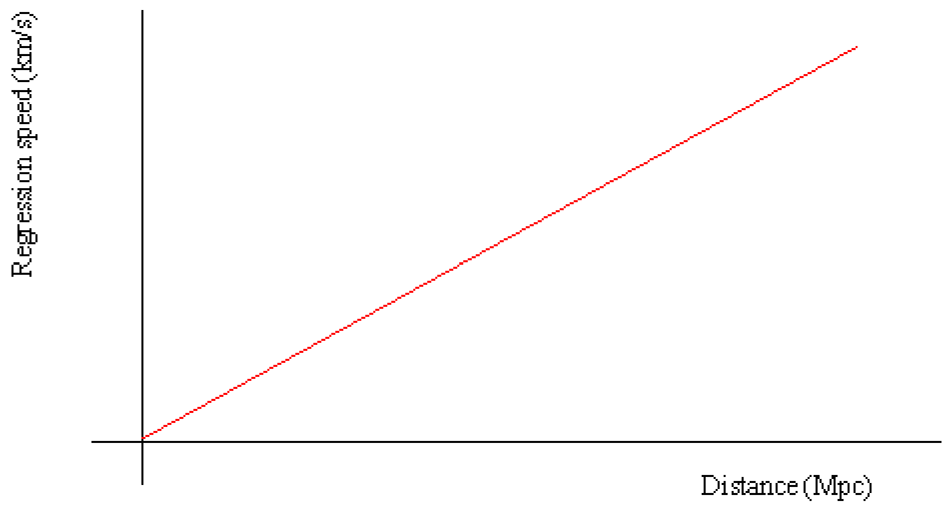


Figure 82 Graph showing Hubble's Law

The equation from this graph is:

$$v = Hd$$ .... *Equation 67*

[v – recession velocity; H – Hubble's Constant; d – distance]

Note that:

- We can work out the recession velocity by using the red shift.
- The distance is worked out by other means which are not needed here. You can look these up in any astrophysics text book.

The constant H is called **Hubble's Constant** and astrophysicists give it the accepted value of **70 ± 30 km s^{-1} Mpc^{-1}**. There is a lot of discussion on the precise value of Hubble's Constant, and there is a lot of uncertainty. The value you are given in the AQA A-level exam is **65 km s^{-1} Mpc^{-1}**. In SI units, $H \approx 2.4 \times 10^{-18}$ s^{-1}. You may well see it written as H_0, as well as H. If you pedantic about it, H is time dependent, while H_0 assumes a constant rate of expansion. I don't think you would lose any marks over this.

More local galaxies do not fit this pattern, because of gravitational interactions. The Andromeda galaxy is actually approaching the Milky Way, and the two will collide. This will not happen for several thousands of millions of years.

Worked Example

The wavelength of a spectral line in the spectrum of light from a distant galaxy was measured at 398.6 nm. The same line measured in the laboratory has a wavelength of 393.3 nm. Calculate:

(a) the speed of recession of the galaxy;

(b) the distance to the galaxy. (c = 3.0 × 108 m s^{-1}, H = 65 km s^{-1} Mpc^{-1})

Answer

(a) Calculate the red shift:

$$\Delta\lambda = 398.6 - 393.3 = 5.3 \text{ nm}$$

$$z = 5.3 \div 398.6 = 0.0133$$

$$v = 0.0133 \times 3.0 \times 10^8 \text{ m s}^{-1} = 3.99 \times 10^6 \text{ m s}^{-1} = \mathbf{4.0 \times 10^6 \text{ m s}^{-1}}$$

This is 4000 km s^{-1}.

(b)

$$d = v/H = 4000 \div 65 = \mathbf{61 \text{ Mpc}}$$

When using Hubble's constant, always remember to convert m s^{-1} to km s^{-1}. You do know how to do that, don't you?

14A.075 The Age of the Universe

There are two possible reasons for red shift in light:

- Stars are moving away from us.
- The universe itself is expanding.

If the second reason were correct, it would seem that the Universe has been much smaller in the past, and that at one point the universe was entirely concentrated in one point. This led to the theory about how the universe began, in a stupendous explosion, the **Big Bang**. (The latter term was first used in a radio broadcast by the Astronomer Royal, Sir Fred Hoyle, in 1953. He believed that the Universe was in a **steady state**, and his use of the term "Big Bang" was dismissive in a sarcastic way.) The **Steady State Theory** says that the Universe was as it is, is, and always will be. The **Big Bang Theory** has considerable credibility among today's astronomers.

If we look at Hubble's graph, we can say that a galaxy at 1000 Mpc is receding at 7000 km/s. By converting Megaparsecs into kilometres we can work out the age of the universe as 4.4 x 10^{17} s, about 14 000 million years. You can try it for yourself.

The age of the universe is given as:

$$\text{Age} = \frac{1}{H_0}$$

.... *Equation 68*

We know that H_0 has a value between 40 and 100 km s^{-1} Mpc^{-1}

At the upper limit of the Hubble constant, the universe is about 10 000 million years old. Studies suggest that the Earth is about half this age.

The simplest model to use to explain how the universe is **expanding** is one you can easily try for yourself. Mark with a felt tip some spots on a balloon. Then blow the balloon up. As it inflates the spots get further apart. As the universe expands, the galaxies move further apart.

There is considerable debate about the fate of the universe:

- Will it keep on expanding and we will be left abandoned in the middle of nowhere?
- Will it bounce back, with everything eventually coming back to where it started from (the **Big Crunch**)?

14A .076 Evidence for the Big Bang

There are a number of pieces of evidence that provide convincing support for the Big Bang:

- The Cosmic Background Radiation.
- Relative abundance of hydrogen and helium.
- Dark Energy

The **Cosmic Background Radiation** was discovered in 1964 by two American radio astronomers, Arno Penzias (1933 - 2024), and Robert Wilson (1936 -). They had constructed a giant horn antenna for use in radio astronomy and satellite communication experiments. The instrument, a **Hogg Antenna** is shown in the picture (*Figure 83*).

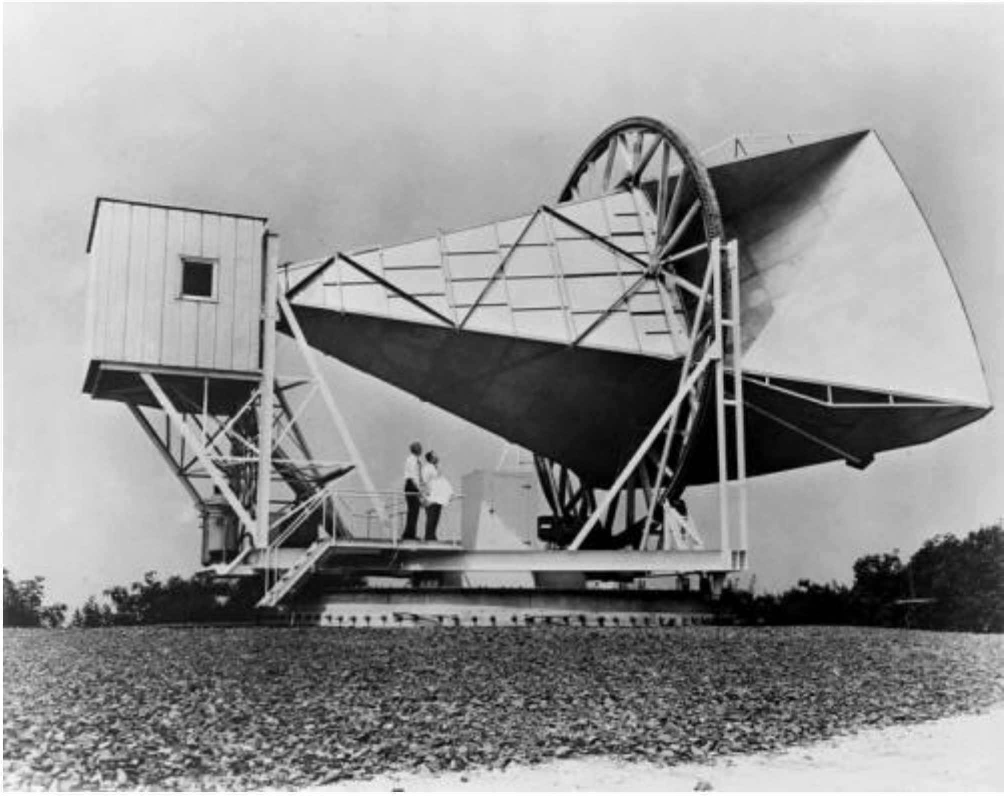


Figure 83 The Hogg antenna used by Penzias and Wilson (NASA, Wikimedia Commons)

Their initial experiments kept on bringing up a microwave signal of wavelength 1.87 μm that corresponded to a temperature of 3 K. Their thoughts were at first that the instrument was not working properly. They checked it over thoroughly and even removed a pigeon's nest. But the signal was still there. After much thought and discussion, the conclusion was that the signal was due to **cosmic background radiation**, predicted in 1948.

This background radiation is universal and gives the Universe a temperature of **2.7 K** (very cold). Things get colder as they expand. If you run a butane (Camping Gaz) stove for about 30 minutes, you will find that it is cold. Energy is taken in from the surroundings to allow the butane to boil and expand as a gas. If you allow compressed air to escape rapidly, the container gets cold. Using the same kind of argument, we can say that the Universe has expanded and was a lot hotter than it is now.

Although the Universe is a very cold place, a steady state theory would suggest that deep space would have a temperature of 0 K.

Stars and galaxies contain **hydrogen and helium in a ratio of 3:1 by mass**. So what?

Helium nuclei have a mass 4 u. A helium nucleus consists of 2 protons and 2 neutrons. There must be 12 hydrogen nuclei, each of 1 u, to fit in with the ratio. A hydrogen nucleus is 1 proton. Therefore, for every two neutrons, there are 14 protons. In other words, the proton to neutron ratio is 7:1. The argument goes that when the universe cooled sufficiently to allow quarks to form in triplets to make baryons, protons formed more readily than neutrons. Calculations involving rest energies of the baryons predict the observed ratio of protons to neutrons of 7:1.

In 1998, supernovae were observed that were much further away than expected. They concluded that the stars must have been accelerating away, and this had been going on for at least 5000 million years. The expectation had been that distant objects would be decelerating as gravitational forces were acting. Many more observations have supported this acceleration. Force has to be applied to get acceleration. Work has to be done to accelerate an object. The energy supplied to do this work is called **dark energy**, but the nature of this is very unclear.

14A .077 Dark Matter and Dark Energy

It is thought that most of the material of Universe consists of **dark matter**. Its key property is that it does NOT interact with the **electromagnetic force**. Therefore, dark matter does not emit, reflect, or absorb electromagnetic radiation. Its presence has been detected by the gravitational effect it has on visible matter. Figures from CERN suggest that 27 % of the universe is dark matter, while only 5 % is matter as we know it.

The evidence for dark matter is that the orbital speeds of stars and galaxies should diminish the further away they are from the centre of a galaxy. Instead, they remain roughly constant (*Figure 84*).

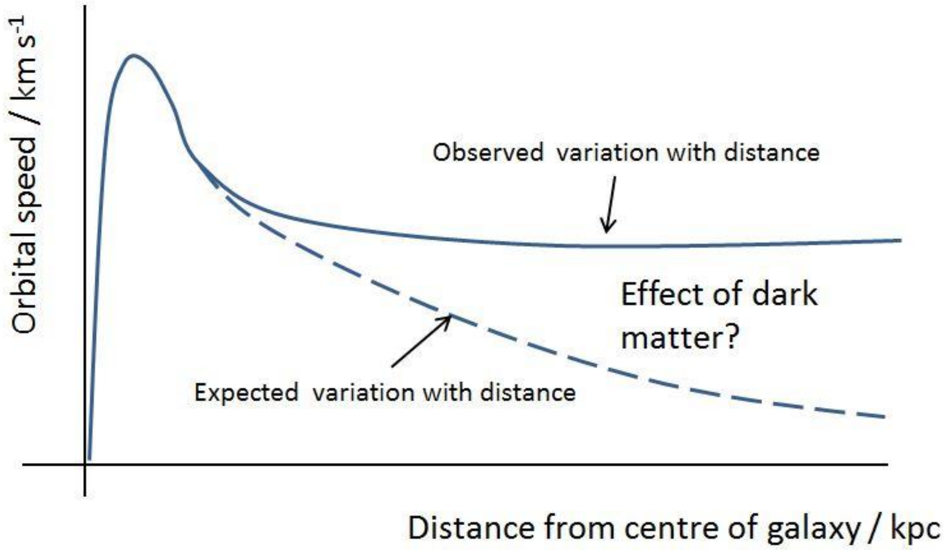


Figure 84 Variation of orbital speed with distance from the centre of a galaxy

The speeds of galaxies within a cluster suggest much more mass than that which is observed.

Gravitational lensing provides evidence (*Figure 85*).

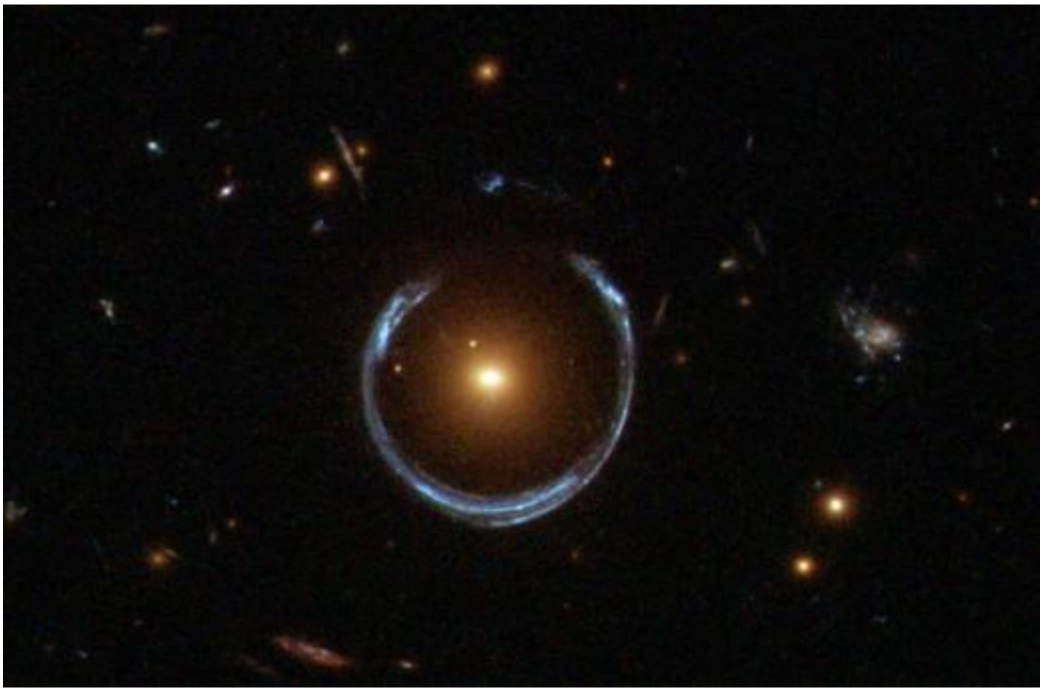


Figure 85 Gravitational lensing (Image from Hubble - Wikimedia Commons)

Models have been used to work out the mass required to give a lensing effect like this. The mass is considerably more than what is observed.

Theoretical physicists are working on the **theory of supersymmetry** in which each known particle has a companion particle, so far undiscovered. For example, **quarks** have particles called **squarks** which are the super-symmetrical partners of the quarks. Similarly, the W boson has a partner called a **wino** (*Figure 86*).

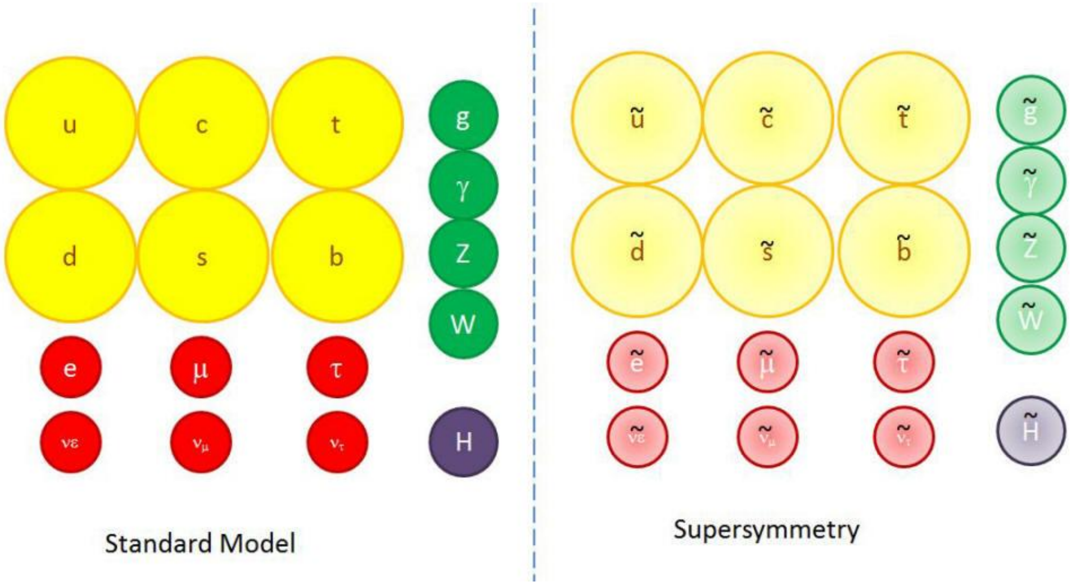


Figure 86 Supersymmetry

Particle physicists are now trying to detect such particles at CERN. The key to detecting them is unexpected results with momentum experiments. So far, no such results have been detected.

The particles in supersymmetry are NOT antiparticles.

Some theoretical physicists think that there is an association between the dark matter and the **Higgs Boson**. The rest energy of dark matter is considered to be between 100 GeV (1×10^{11} eV) and 1 TeV (1×10^{12} eV). The Higgs Boson has a rest energy of 125 GeV and is an elementary particle. When the Universe immediately after the Big Bang was still extremely hot, theoretical physicists suggest that matter and dark matter were balanced in thermal equilibrium. Since it is believed that matter is derived originally from the Higgs

Boson, it makes sense that dark matter is as well. It gave rise to a range of particles known as **WIMPS** (weakly interacting massive particles). The weak part refers to the **weak interaction**.

Some physicists think that **Dark Energy** is connected with the Higgs Boson, but others disagree.

Dark energy makes up 68 % of the universe. It is associated with the vacuum of space. It is described as being evenly distributed in both space and time and is considered to provide the repulsive force that is causing the universe to expand.

14A.078 Timeline of the Big Bang

Much of what went on in the Big Bang happened in the **first second** - rather less than the time you took to read this. The timeline is presented in the table below:

Time	Name	Size of the universe	Temperature / K	What happened
0	The Big Bang	0	∞	All matter in the Universe was confined to an infinitesimally small space.
0 - 10^{-43} s	Pre-Planck Era	<10^{-35} m	>10^{32}	Nobody is sure. All fundamental forces are unified.
10^{-43} – 10^{-36} s	Grand Unification Epoch	10^{-35} m	10^{32}	Gravity separates from the other fundamental forces. Earliest elementary particles and antiparticles.
10^{-36} – 10^{-32} s	Inflationary epoch	0.1 m	10^{27}	The strong force separates. This causes massive expansion of the Universe, which consists of a hot quark-gluon plasma.
10^{-32} – 10^{-12} s	Electroweak epoch	10^{10} m	10^{22}	Exotic particles come into existence, for example W, Z, and Higgs Boson.

10^{-12} – 10^{-6} s	Quark epoch	10^{15} m	10^{21}	Quarks start to form. Fundamental forces take their present form. Matter and antimatter particles annihilate. There is a small surplus of matter. Some sources suggest that the Universe temperature increased markedly (by about 100 000 times)
10^{-6} - 1 s	Hadron Epoch	10^{17} m	10^{18}	Protons form. Electrons colliding with protons are captured to form neutrons. Neutrinos are formed, travelling at the speed of light. Protons combine with electrons to form hydrogen atoms.
1 s - 3 min	Lepton epoch	10^{18} m	10^{10}	With the annihilation of protons and antiprotons, there are many high energy photons, which collide to cause pair production of electrons and positrons.
3 - 20 min	Nucleosynthesis	10^{20} m	10^{9}	In this period, the temperature is sufficiently high to allow the fusion of protons and neutrons to form the first very simple nuclei, like hydrogen, helium, and lithium. After 20 minutes, the temperature has fallen to too low a value to allow fusion to happen.
20 min - 240000 y	Photon Epoch	10^{24} m	10^{9}	The Universe is a hot opaque soup of plasma. Leptons and antileptons are annihilating to produce photons. These interact frequently with protons, electrons, and nuclei.

240000 - 300000 y	**Recombination** and decoupling	10^{25} m	3000	The nuclei are now combining with electrons to form atoms. Small clumps of material start to gather. As electrons combine with nuclei, the universe starts to become transparent to light. Photons are no longer interacting with protons and electrons. These are released (**decoupled**) to form the Cosmic Background Radiation.
300000 - 150 $\times 10^6$ y	Dark Ages	10^{26} m	60	The Universe is dark, as the first stars are only just starting to be formed. There is little activity, and the Universe is dominated by dark matter.
150 - 1000 $\times$ 10^6 y	Reionisation	10^{26} m	19 K	The first quasars are formed by gravitational collapse. This causes ionisation in much of the surrounding universe. Most of the universe is ionised again to form plasma,
1000 - 5000 $\times$ 10^6 y	Star and Galaxy formation	10^{27} m	4 K	Small clumps of cosmic gas coalesce under the influence of gravity. As the material gathers, its temperature rises sufficiently to cause fusion. Stars a born. These stars a metal-free and are usually large and short-lived. They form supernovae that produce many heavy elements from their explosions. Large volumes of matter tend

				to gather together to form galaxies
13.7×10^9 y	Present	10^{27} m	2.7	The expansion of the universe continues and stars are recycled. The furthest objects that can be observed are the cosmic background radiation photons.

When the Big Bang occurred, it is assumed that all **space time** was crammed into that infinitesimally small space. The explosion did not propagate into a pre-existing vacuum. Nor did the space time lead the explosion. Space-time expanded with everything else in the universe at the same rate.

14A.079 Quasars

Quasi-stellar objects (**quasars**) were discovered in 1960. They are very luminous objects at immense distances. They appear to light telescopes as stars but are not typical:

- They outshine complete galaxies.
- Spectra show lines that correspond to no known elements.
- However, the lines were in fact considerably red-shifted.
- Some are intense radio sources.

The red shift suggests that the objects are moving away from us at 15 % of the speed of light. According to Hubble's Law, that means that they are a very long way away. So we can say that Quasars are:

- very distant.
- very bright.
- smaller than a galaxy.

Here is a picture of one (*Figure 87*).

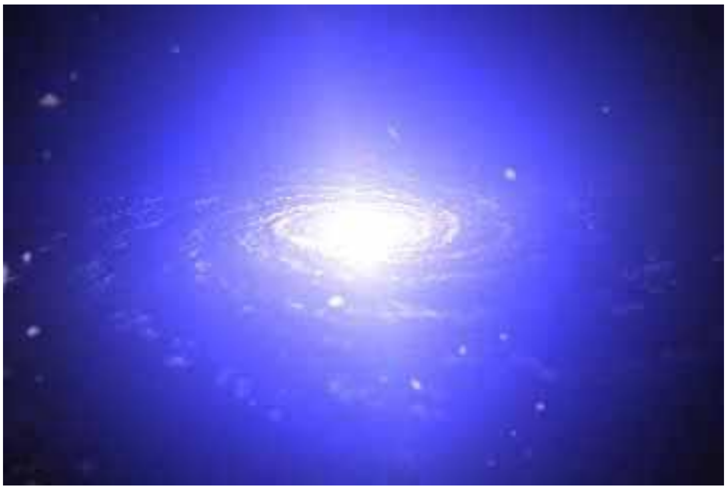


Figure 87 A quasar

Nobody is quite sure what they are, but the predominant belief that:

- They are **super massive black holes** about 10^6 to 10^9 times the solar mass.
- They gobble up stars at about 10 solar masses every year, i.e. a giant cosmic Hoover.
- The dense flow of matter can force jets of matter to stream away from the disc.

Computer modelling suggests a structure like this (*Figure 88*).

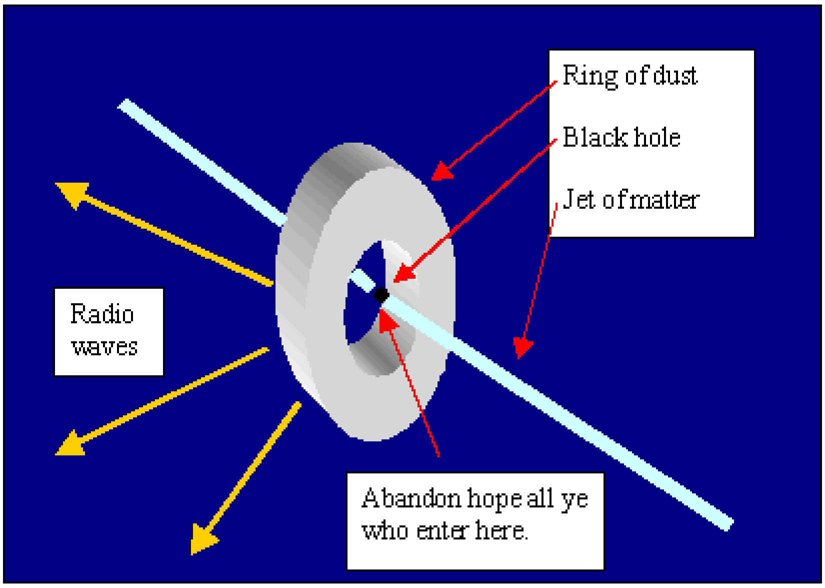


Figure 88 Structure of a quasar

A stylised painting shows what a quasar would look like from a relatively safe few hundred light years (*Figure 89*).

Figure 89 A stylised picture of a quasar

What you see will depend on your viewing point:

- from the side you see a **radio galaxy.**
- closer to the line of the jet you see a quasar.
- in the line of the jet, you see a **blasar** (an extremely luminous galactic object.)

Some astronomers believe that there are quasars closer to home, in nearby galaxies. However, the accepted belief is that they are distant objects. Therefore, they were around very early in life of the universe. The furthest yet observed is quasar 0051 279 discovered in 1987. It is receding at 93 % of the speed of light and is 13 100 million light years away.

Being gobbled up by a super massive black hole is not yet on the agenda, as both the Milky Way and Andromeda are relatively inactive galaxies, with few stars near the supermassive black holes at the centre.

14A.0710 Critical Density (*Welsh Board*)

We have considered the origins of the Universe in the Big Bang, but we have not thought about the **fate** of the Universe. We know that the Universe is currently expanding. What is going to happen to it? Will it eventually stop expanding, and fall back on itself (the Big Crunch)? Will the expansion eventually be halted and the Universe stays where it is? Or will it continue to expand (even if at an ever decreasing rate)?

In the **Theory of General Relativity**, Einstein argued that gravity has the effect of curving the surrounding space. This is described in Topic 15. The fate of the Universe is determined by the **density** of the matter that is in it.

- If the density of the matter in the Universe is high, we have a **closed Universe** that can be modelled as a sphere. The gravity of the Universe will slow the expansion down and eventually start to pull all the material back together. In the closed universe, parallel rays of light will eventually **meet** at some extremely distant point.
- If the density of the matter in the Universe is low, we have an **open Universe**, the gravitational forces are insufficient to stop the expansion. The Universe will continue to expand. Parallel rays of light will **diverge**. This can be modelled as the hyperbolic geometrical shape.
- If the density is just right, we have a **flat Universe**. Here the parallel rays of light remain **parallel**. The expansion stops at an **infinite** time.

  We can show this as a schematic (*Figure 90*).

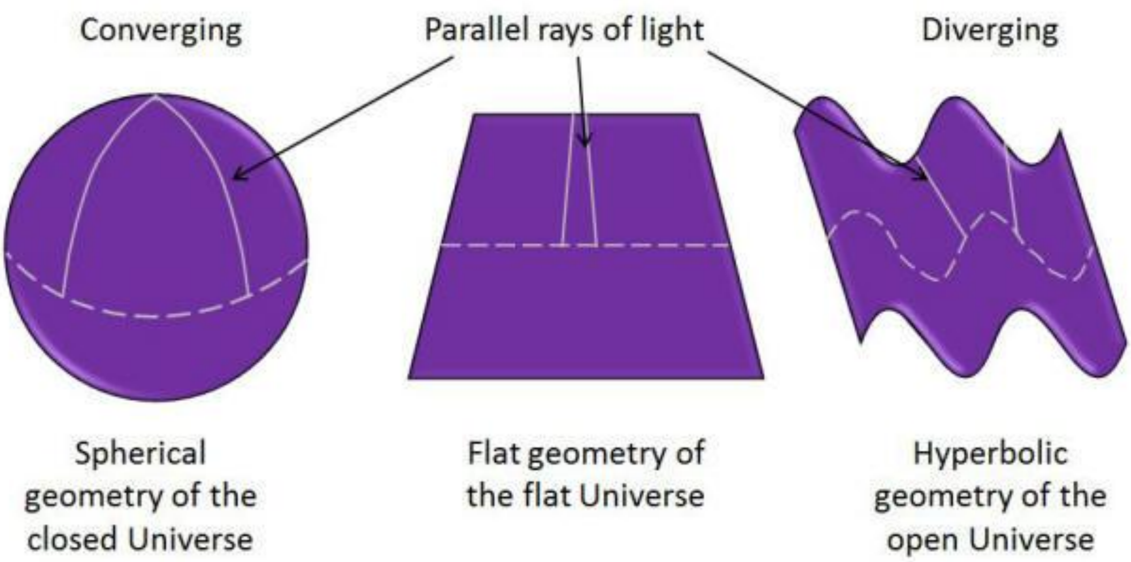


Figure 90 Possible models of the Universe

The density to give rise to the flat Universe is called the **critical density**. The critical density is given by the equation:

$$\rho_c = \frac{3{H_0}^2}{8\pi G}$$

.... *Equation 69*

where:

- H_0 is Hubble constant. In SI units, $H_0 = 2.2 \times 10^{-18}\ \text{s}^{-1}$.
- G is the gravitational constant = $6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$.

You must use the SI units for Hubble's Constant in this case.

If you use the Hubble constant H_0 as 70 km s^{-1} Mpc^{-1}, it will give you an answer of 9.0×10^{12} kg m^{-3}. This is the density of a black hole.

Derivation

Consider the Universe as a uniform hollow sphere of mass m in which every particle is evenly spread throughout, whether it is from dust, stars, planets, or dark matter. It forms a material of uniform density of a very low value, and has a mass, M. The sphere is expanding at a speed v (*Figure 91*).

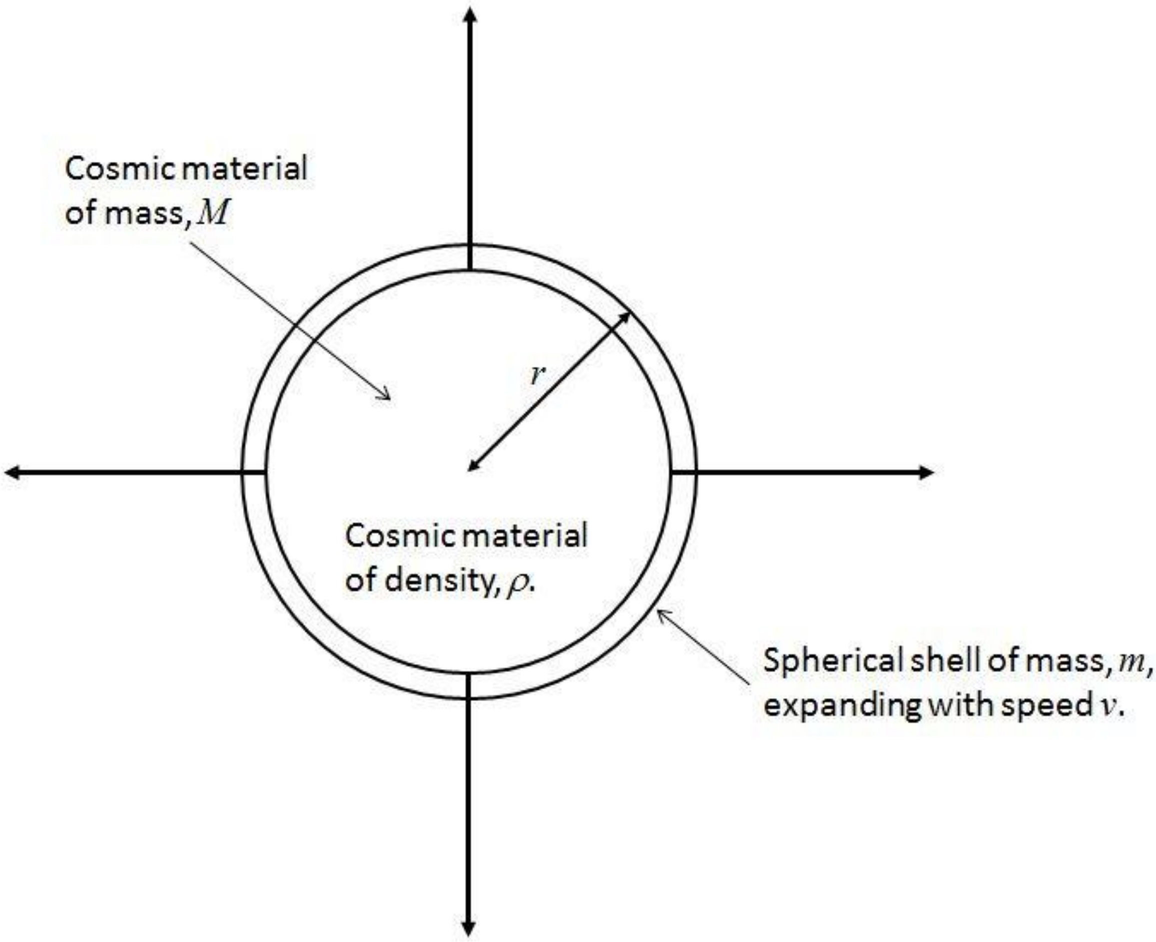


Figure 91 Universe expanding as a sphere

The material experiences a gravitational force inwards. We can ignore the gravitational effects of the material outside the shell (if there is any) as its extent is infinite, and any pull to one side is cancelled out by the pull to the other side.

We can write an expression for the mass (= density × volume):

$$M = \frac{4}{3}\pi r^3 \rho$$

.... *Equation 70*

From our studies of gravitational fields, we know that the gravitational potential energy, E_{g}, is given by:

$$E_{\mathrm{g}} = (-)\frac{GMm}{r}$$

.... *Equation 71*

Strictly speaking we should use the minus sign, but we will keep E_{g} positive in this argument.

We substitute for M:

$$E_{\mathrm{g}} = \frac{4Gm\pi\rho r^2}{3}$$

.... *Equation 72*

Notice that the r term downstairs has cancelled with the r^3 term upstairs to leave r^2 upstairs.

The shell is expanding. The total kinetic energy is given by the usual:

$$E_k = \frac{1}{2}mv^2$$

.... *Equation 73*

From our studies of the Hubble constant, we know that:

$$v = H_0 r$$ .... *Equation 74*

So, we can substitute for v and write:

$$E_k = \frac{mH_0{}^2r^2}{2}$$

.... *Equation 75*

The definition of potential energy is the work done to move the object to infinity. At this point the potential energy is zero. By the Law of Conservation of Energy, the kinetic energy is also zero, so we can equate the two equations for potential and kinetic energy to give:

$$\frac{4Gm\pi\rho r^2}{3} = \frac{mH_0{}^2r^2}{2}$$

.... *Equation 76*

We can see that the r^2 and the m terms cancel:

$$\frac{4G\pi\rho}{3} = \frac{H_0{}^2}{2}$$

.... *Equation 77*

The density term ρ is the **critical density**, which we will call ρ_c. Now we can rearrange to make ρ_c the subject to give us:

$$\rho_c = \frac{3H_0{}^2}{8\pi G}$$

.............*Equation 78*

The value of H_0 is in the range of $1.6 \times 10^{-18}\ s^{-1}$ to $3.2 \times 10^{-18}\ s^{-1}$.

Questions

Tutorial 14A.07

14A.07.1

A star is moving away from the Earth at 5000 km s^{-1}. A certain wavelength has been detected in its spectrum which corresponds to a line of wavelength 350 nm as measured in a laboratory. What is the wavelength of this line?

14A.07.2

Venus has a diameter of 12 200 km and a rotational period of 243 days.

(a) What is its angular velocity and its linear speed of rotation at the equator.

(b) Radio waves of wavelength 1.0 m are used to determine the speed of rotation. What is the expected shift in wavelength reflected at opposite edges of the equator?

14A.07.3

A binary star system is studied where it is concluded that both stars are of the same mass. Their orbital period is 2.4 years. A certain element is known to give a spectral line of 460 nm. And this is observed at time zero. 0.6 years later, the same line is observed to be at 459.92 nm. At the same time another spectral line is seen.

(a) Where is the second spectral line seen?

(b) Explain the observation.

(c) What is the orbital speed of the star?

(d) What is the radius of the orbit?

(1 year = 365.25 days)

14A.07.4

A distant galaxy has a red shift of 15 %.

(a) What is its speed of recession?

(b) If H_0 has a value of 100 km s^{-1} Mpc^{-1}, what is its distance?

14A.07.5

H_0 has a value between 40 and 100 km s^{-1} Mpc^{-1}.

What is the age of the universe using these extremes of value? 1 pc = 3.086×10^{16} m

14A.07.6

Show that the critical density is about 10^{-26} kg m^{-3}.

14A.07.7

Use your answer to question 6 to work out how many hydrogen atoms there are in every cubic metre.

Mass of Hydrogen atom = 1.67×10^{-27} kg.

Answers to Questions

Tutorial 14A.01

14A.01.1

Focal length = $1/P$= 1/0.2 D = **5 m**

14A.01.2

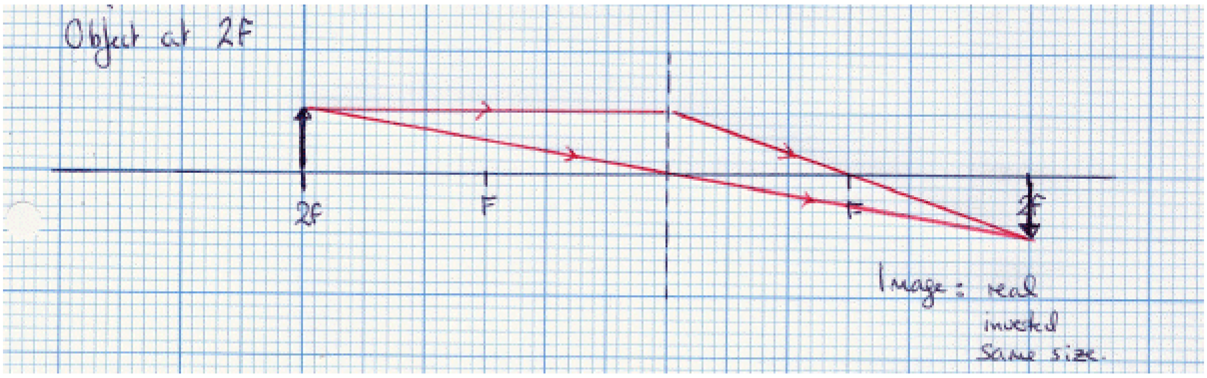


Image is real, inverted, and the same size.

14A.01.3

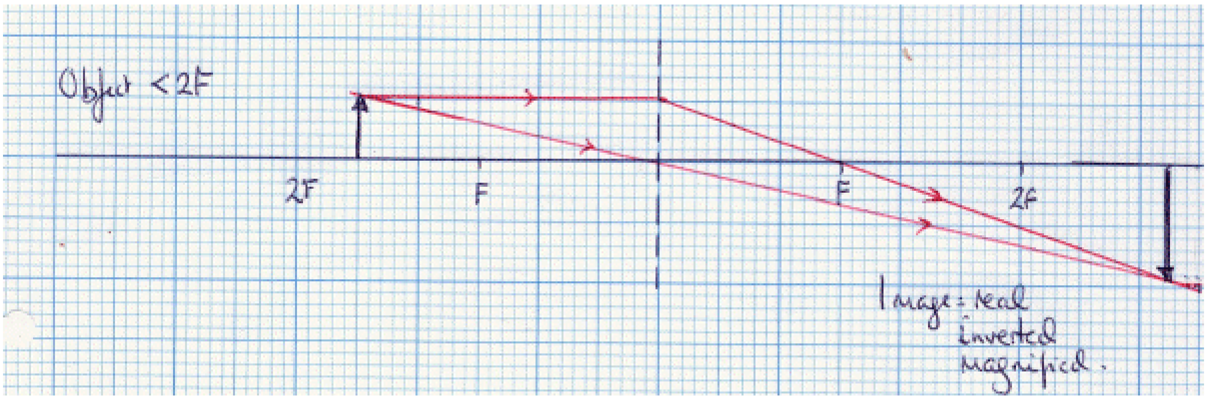


Image is real, inverted, and magnified.

14A.01.4

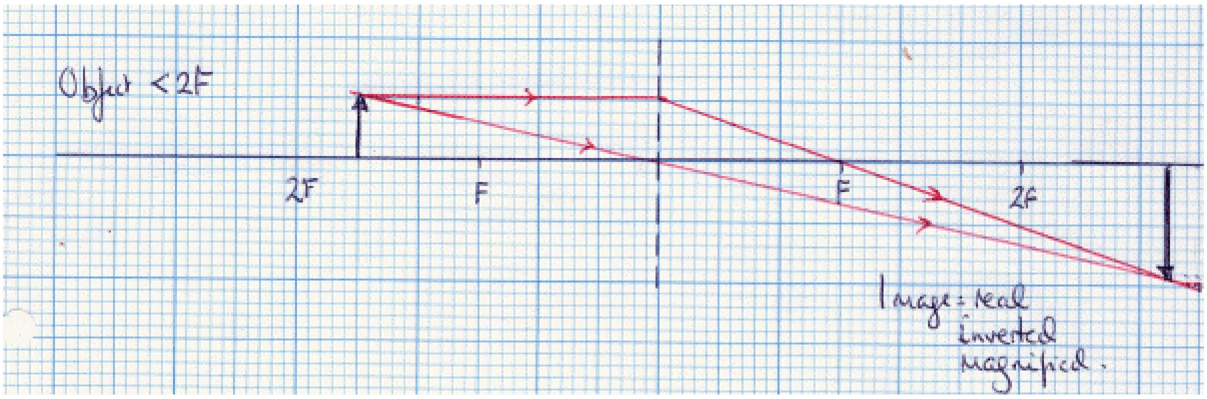


Image is real, inverted, and magnified.

14A.01.5

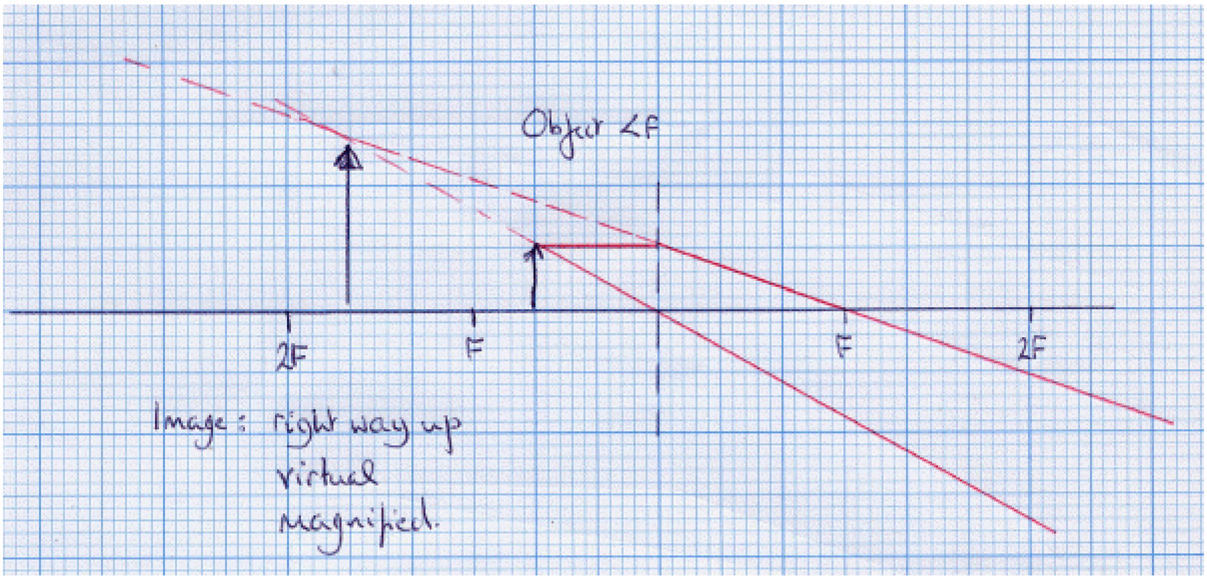


Image is virtual, right way up, and magnified.

You will often find the term **erect** used for right way up. (After teaching this stuff for thirty years, the old joke wears a bit thin...)

14A.01.6

$$1/f = 1/u + 1/v$$

1/40 cm = 1/20 cm + 1/v

1/v = 0.025 cm^{-1} - 0.05 cm^{-1} = -0.025 cm^{-1}

v = - 40 cm

magnification = v/u = -40/20 = **(-)2**

Therefore, the image is 4.4 cm across.

The image is virtual, magnified, and erect (right way up).

14A.01.7

Magnification = f_o/f_e = 220 cm ÷ 2 cm = 110

Angle subtended by the Moon = 8.8 × 10^{-3} rad × 110 = **0.97 rad**

Tutorial 14A.02

14A.02.1

(a)

The light does not have to pass through glass, so will not suffer any chromatic aberration. All colours are reflected by the same amount.

(b)

The region lost by the presence of the secondary mirror is insignificant. It would not matter if the dead space caused by the secondary mirror were removed.

14A.02 2

For small angles in radians, $\sin\theta = \tan\theta = \theta$.

Angle of moon = 3500 km ÷ 400000 km = **8.8×10^{-3} rad**

1 rad = $360/2\pi$ = 57.3°

Angle in degrees = **0.504°** (= 30' 15'')

14A.02.3

For small angles in radians, $\sin\theta = \tan\theta = \theta$.

Angle of moon = 3500 km ÷ 400000 km = 8.8×10^{-3} rad

Diameter of the Sun = 150×10^{6} km × 8.8×10^{-3} rad = 1.32×10^{6} km

14A.02.4

Use $\theta = \lambda/D$ = 600×10^{-9} m ÷ 0.15 m = **4×10^{-6} rad**

Tutorial 14A.03

14A.03.1

The diameter of the Puerto Rico instrument is four times that of Jodrell Bank. Therefore its power is 16 times.

14A.03.2

The Puerto Rico instrument is static, so it cannot track objects.

It sweeps across the sky as a result of the Earth's rotation.

14A.03.3

$$\theta = \frac{\lambda}{D}$$

θ = 10 m ÷ 75 m = **0.13 rad** (= 7.6°)

Tutorial 14A.04

14A.04.1

10 pc = 3.086×10^{17} m = 3.086×10^{14} km

Time taken for supersonic plane = 3.086×10^{14} km ÷ 3000 km/h

= **1.02×10^{11} h** (= 11 million years)

14A.04.2

Difference = dimmest - brightest = 0.50 - -1.46 = 1.96

Ratio = $2.512^{1.96}$ = **6.08**

Sirius is 6.08 times brighter than Betelgeuse.

14A.04.3

It's not a fair test, because some stars are a lot further away than others.

Therefore, a bright star at a long distance may appear dimmer than a comparatively dim star, which is closer to us.

14A.04.4

Distance to Sun from Earth in pc = 1.496×10^{11} m ÷ 3.086×10^{16} m

$d = 4.85 \times 10^{-6}$ pc

$-26.7 - M = 5 \times \log (4.85 \times 10^{-6} \text{ pc}/10 \text{ pc})$

$-M = (5 \times -6.314) + 26.7 = -31.57 + 26.7 = -4.87$

M = **+4.87**

The Sun has a higher positive magnitude than Alpha Centauri, so it is slightly dimmer.

14A.04.5

(i)

Distance to Bellatrix from Earth in pc = 470 ly ÷ 3.26 ly/pc

d = **144 pc**

(ii)

m - -4.2 = 5 × log (144 pc/10 pc)

m = (5 × 1.158) - 4.2 = 5.79 - 4.2 = **+1.6**

(iii)

At 10 pc Elinath would appear dimmer as its magnitude is less negative.

Since it appears to be as bright as Bellatrix, it must be closer to the Earth

Tutorial 14A.05

14A.05.1

T = 0.00289 m K ÷ 600 × 10^{-9} m

T = **4800 K**

14A.05.2

Energy change = -1.51 eV - -3.41 eV = 1.90 eV

Energy change (J) = 1.90 eV × 1.60 × 10^{-19} J eV^{-1} = 3.04 × 10^{-19} J

$\lambda = hc/E$ = 6.63 × 10-34 J s × 3.0 × 10^{8} m s^{-1} ÷ 3.04 × 10^{-19} J = **6.54 × 10^{-7} m** = 654 nm

14A.05.3

T = 0.00289 m K ÷ 6.54 × 10^{-7} m = **4420 K**

14A.05.4

You would see little evidence of Balmer Lines.

You would see spectral lines for iron and calcium.

You would also see evidence for molecules of titanium dioxide.

14A.05.5

(a)

Electrons are excited when atoms collide. They rise to a higher energy level by a discrete amount. As they fall to lower energy levels, they emit photons of energy corresponding to the energy difference between the energy levels.

Absorption lines against a continuous spectrum are seen as the photons emitted at particular wavelengths are emitted in all directions, so would produce a dark line.

(b)

The largest transition would give a photon of the shortest wavelength. This would be from $n = 6$ to $n = 2$.

(c)

This is at 410 nm

(d)

The lines are called the Balmer series.

(e)

Classes A, B and F

14A.05.6

We use:

$$P = 4\pi r^2 s T^4$$

$$P = 4 \times \pi \times (6.96 \times 10^{8}\ \text{m})^2 \times 5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4} \times (6000\ \text{K})^4$$

P = **4.47 × 10^{26} W**

The power per unit area = 4.47 × 10^{26} W ÷ 6.09 × 10^{18} m^2 = **7.34 × 10^{7} W m^{-2}**

Peak wavelength λ_{max} = 0.00289 m K ÷ 6000 K = **4.82 × 10^{-7} m** = 482 nm

14A.05.7

$$\Delta m = 2.5 \lg\left(\frac{I_A}{I_B}\right)$$

Δm = 4.8 - -0.2 = 5

5 = 2.5 × $\log (I_P/I_S)$

$\log (I_P/I_S)$ = 2

(I_P/I_S) = 100

PLC A2.47 is 100 times more powerful than the Sun.

14A.05.8

(a)

Area of Sun = 4 × π × $(0.7 \times 10^9)^2$ = **6.16×10^{18} m^2**

(b) Use:

$$\frac{P_P}{A_P {T_P}^4} = \frac{P_S}{A_S {T_S}^4}$$

$$\frac{4.0 \times 10^{28}\ \text{W}}{A \times (5500\ \text{K})^4} = \frac{4.0 \times 10^{26}\ \text{W}}{6.16 \times 10^{18}\ \text{m}^2 \times (5800\ \text{K})^4}$$

$$A = \frac{4.0 \times 10^{28}\ \text{W} \times 6.16 \times 10^{18}\ \text{m}^2 \times (5800\ \text{K})^4}{4.0 \times 10^{26}\ \text{W} \times (5500\ \text{K})^4} = 7.61 \times 10^{20}\ \text{m}^2$$

$r^2 = 7.61 \times 10^{20}\ \text{m}^2 \div 4\pi = 6.06 \times 10^{19}\ \text{m}^2$

$r = 7.80 \times 10^9$ m

D = **1.56×10^{10} m**

(c)

1.56×10^{10} m ÷ 1.50×10^9 m = **10.3** = **10 times** the diameter of the Sun

Tutorial 14A.06

14A.06.1

Star	Luminosity (Sun = 1)	Surface Temp (K)	Group
Sun	1.0	5800	Main sequence
Betelgeuse	20 000	3000	Red Supergiant
Aldebaran	200	4700	Giant
Regulus	200	14000	Main Sequence
Rigel	20000	13000	Main Sequence
Sirius B	0.002	20000	White Dwarf

14A.06.2

The process was much more rapid. The temperature rises from 2500 K to 10000 K over a period of a few thousand years.

The temperature rise from 10 000 K to 20 000 K takes about 9000 years.

14A.06.3

1 pc = 3.26 ly

320 pc = **1040 ly**

14A.06.4

Find the volume:

$$V = 4/3\pi r^3.$$

V for Earth = $4/3 \times \pi \times (6.4 \times 10^6 \text{ m})^3 = 1.10 \times 10^{21} \text{ m}^3$

Density = mass/volume = 6.0×10^{24} kg ÷ 1.10×10^{21} m^3 = **5500 kg m^{-3}**.

For the Sun:

$V = 4/3 \times \pi \times (7.0 \times 10^8 \text{ m})^3 = 1.44 \times 10^{27} \text{ m}^3$

Density = mass/volume = 2.0×10^{30} kg ÷ 1.44×10^{27} m^3 = **1400 kg m^{-3}**.

For the white dwarf:

Density = mass/volume = 2.0×10^{30} kg ÷ 1.10×10^{21} m^3 = **1.8×10^9 kg m^{-3}**.

14A.06.5

$$g = -\frac{GM}{r^2} = -\frac{6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2} \times 6 \times 10^{24}\ \text{kg}}{(200\ \text{m})^2} = 1.0 \times 10^{10}\ \text{N kg}^{-1}$$

$$v = \sqrt{(2gr)} = \sqrt{(2 \times 1.0 \times 10^{10} \times 200)} = \mathbf{2 \times 10^6\ m\ s^{-1}}.$$

14A.06.6

(i)

The Event Horizon is the boundary at which light cannot cross.

(ii)

Use the equation

$$R_S = \frac{2GM}{c^2}$$

$$R_S = \frac{2 \times 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2} \times 2.0 \times 10^{36}\ \text{kg}}{(3.0 \times 10^8\ \text{m s}^{-1})} = \mathbf{2.96 \times 10^9\ m}$$

Tutorial 14A.07

14A.07.1

Use:

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

$\Delta\lambda$ = - -5000 × 10^3 m s^{-1} × 350 × 10^{-9} m ÷ 3 × 10^8 m s^{-1}

$\Delta\lambda$ = 5.83 × 10^{-9} m

As the star is receding, we add the difference to the wavelength:

New wavelength = **355.83 nm**

14A.07.2

(a)

Angular velocity = $2\pi/t$ = 2 × π ÷ (243 dy × 86400 s) = 2.99 × 10^{-7} rad s^{-1}

Linear speed at equator = ωr = 2.99 × 10^{-7} rad s^{-1} × (12 100 × 10^3 m / 2) = 1.81 m s^{-1}.

(b) Use:

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

$\Delta\lambda$ = - -1.81 m s^{-1} × 1.0 m ÷ 3 × 10^8 m/s

$\Delta\lambda$ = **6.03 × 10^{-9} m** = 6.03 nm

14A.07.3

(a)

460.08 nm

(c)

At time zero the stars are in line with the observer. At 0.6 y (1/4 orbit), one star is moving away, and one is moving towards the observer.

(c)

$\Delta\lambda$ = 0.08 nm

z = 0.08 nm ÷ 460 nm = 1.74×10^{-4}

$v = zc = 1.74 \times 10^{-4} \times 3.0 \times 10^{8}$ m s^{-1} = **5.22×10^{4} m s^{-1}**

(d)

$$r = \frac{vT}{2\pi} = \frac{5.22 \times 10^{4} \text{ m s}^{-1} \times 2.4 \text{ y} \times 365.25 \text{ d y}^{-1} \times 86400 \text{ s d}^{-1}}{2\pi} = \mathbf{6.3 \times 10^{11} \text{ m}}$$

14.07.4

(a) Use:

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

0.15 = - -v ÷ 3 × 10^{8} m s^{-1}

$v = 4.5 \times 10^{7}$ m s^{-1} = **45 000 km s^{-1}**

(b)

$d = v/H$ = 45 000 km s^{-1} ÷ 100 km s^{-1} Mpc^{-1} = **450 Mpc** (which is quite a long way)

14A.07.5

1 Mpc = 3.086×10^{16} m × 10^{6} = 3.086×10^{22} m

1 Mpc = **3.086×10^{19} km**

For the lower limit:

H = 40 ÷ 3.086×10^{19} km = 1.30×10^{-18} s^{-1}

Age = $1/H$ = **7.71×10^{17} s**

(1 year = 3.15×10^{7} s

Age = 24 000 million years)

For the upper limit:

H = 100 ÷ 3.086×10^{19} km = 3.24×10^{-18} s^{-1}

Age = $1/H$ = **3.086×10^{17} s**

Age = 9800 million years

14A.07.6

Formula:

$$\rho_c = \frac{3{H_0}^2}{8\pi G}$$

ρ_c = (3 × (2.2×10^{-18} s^{-1})^2) ÷ (8 × π × 6.67×10^{-11} N m^2 kg^{-2})

= 8.7×10^{-27} kg m^{-3} = **0.9×10^{-26} kg m^{-3}**.

14A.07.7

Number of H atoms for each cubic metre = 8.7×10^{-27} kg m^{-3} ÷ 1.67×10^{-27} kg = 5.21 m^{-3}.

But we can't have 0.21 of a hydrogen atom, so there are 5 hydrogen atoms every cubic metre.